STRESS ANALYSIS OF A SHELL STRUCTURE

by 884

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SYNOPSIS

The formulas for membrane forces and displacement in the parabolical dome roof and the conical shell wall which are loaded symmetrically with respect to the axis are derived, and the differential equations of bending for the parabolical dome roof are simplified from the general equation of shells which was derived by Timosenko[1]. While the equation for the conical shell wall is derived from the concept of the beams on elastic foundation[3].

These equations are difficult to solve, hence the finite difference procedure is used[4]. Thus the problem is reduced to the simple task of solving a system of simultaneous linear algebriac equations. The numerical computation involved in the procedure is considerably simplified by two devices. First, the number of equations necessary to attain sufficient accuracy is reduced by an evaluation of the error introduced in substituting central differences for derivatives. Secondly, the solution of simultaneous equations is determinated by using the digital computer.

The consistent correction forces at the edge of the dome, ring, and the conical shell wall are computed according to the compatibility equations. By superposition of the forces found by the membrane and bending theory, the total forces acting on the shell can be obtained.

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* Numbers on the upper right corner of the sentences refer to reference listed in bibliography.

INTRODUCTION

During the last twenty years the shell structure has achieved extraordinary practical importance. The main reason is not for beautiful forms but for the characteristic interplay of force in spatial surface sturctures, which results in a considerable saving in building cost.

The dome is one kind of shell. Many massive domes, from those of the pantheon (Fig. 1) and St's Peter's (Fig. 2) to those of the auditorium in the university of Illinois (Fig. 3) were built as shells. If the menthod of raising a dome with a balloon as the form work is successfully developed, the choice of a dome roof and floor will be the most economical structure in building construction.

Morden dome thickness is small compared with the other dimensions. The laws governing this interplay of forces connot be explained by the elementary single dimension stresses analysis of linear members, mathematically elaborate shell theory has been developed. However because of the some mathematical difficulties, the practical design of these shells has been only based on the assumption of a membrane state of stress in the shell.

Timoshenko developed the general equations for an axisymmetric shell which carries no surface loading, but only edge moments and shears. These equations are homogeneous. To solve a shell problem a membrane solution has to be superimposed upon the solution to the homogeneous equations and the constant of integration are adjusted to suit the boundary condition. But the general equation of the snell is difficulty to integrate. Approximate simplified analysis only is derived for the spherical dome and circular cylinder of constant thickness. Therefore the finite difference will be

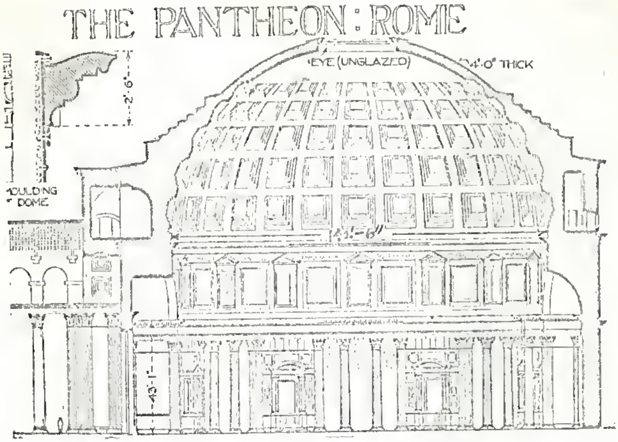

Fig. 1 The Patheon, Rome, AD 120-124, Sect. thro'.
Portic & Rotunda

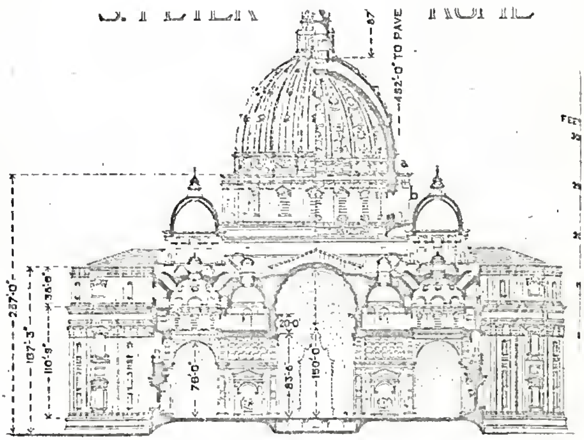

Fig. 2 S. Peter, Rome, AD 1506-1626

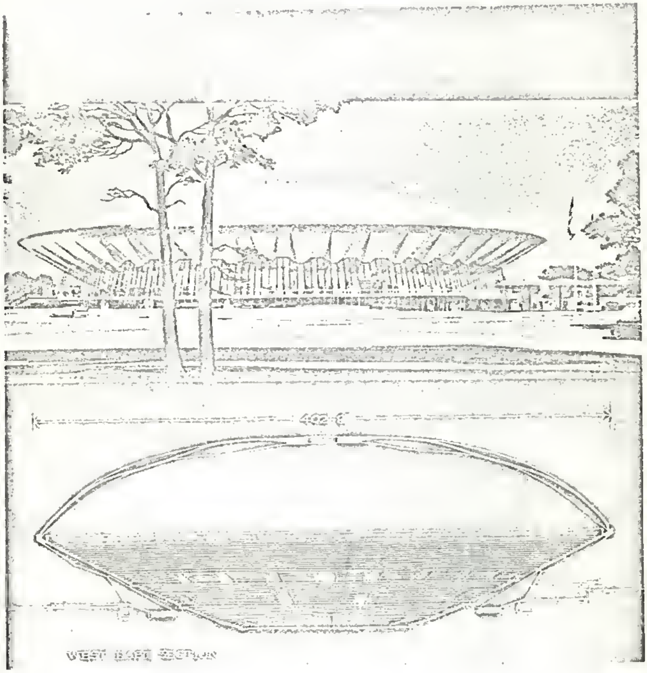

Fig. 3 The auditorium of the Uni. of Ill.

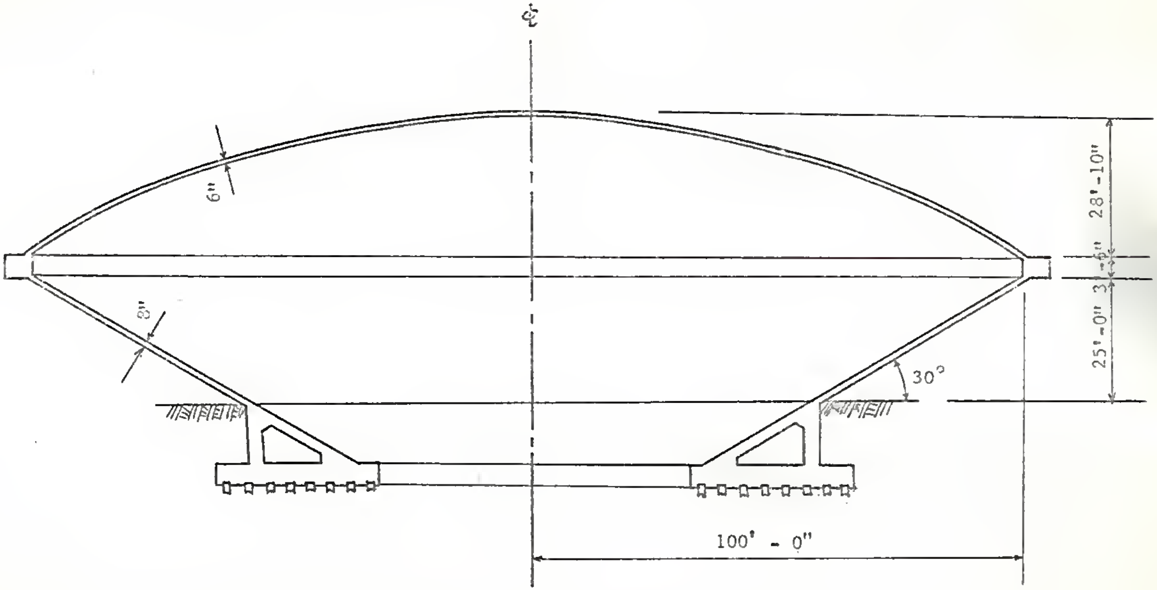

Fig. 4 Central section of the structure

applied to solve this particular form of the dome roof and shell wall in this report.

The dome structure which is proposed in this report is a parabolical dome roof with a conical shell wall (Fig. 4). These two shells joined by a tension ring. The top radical partterned dome, like a cover on a soup tureen. This type of form can be used as gymnasium, Auditorium, theatre, exhibition building, etc. The slop of the wall can be constructed as step down seating.

Both part of the shells are constant in thickness; it is six inches for the dome roof and eight inches for the shell wall, with two hundred feet in free-span diameter. The ring encircling the edge of the dome and wall is provided to take the horizontal thrust; the conical shell wall is assumed to be fixed on the top of the foundation. For ease in checking, numerical calculations follow the formula derived in each part.

DEFINITIONS AND GENERAL MEMBRANE THEORY FOR SHELLS OF REVOLUTION

Definitions

A thin shell is a curved slab, its thickness is small in comparison with the other dimensions of the shell and with its radii of curvature, the surface that bisects the thickness of plate is called the middle surface.

Domes are defined as thin shells in the form of surfaces of revolution. The parabolical dome roof and the conical wall in this report, the surface is described by revolving an arc of a circle. The center of the circle is on the axis of rotation.

Membrane theory for shells of revolution

Basic assumption

1. Bending of the shell is negligible.

2. Middle surface of the shell can be assumed to suffer only extension, and a pure membrane state of stress exist. (Shearing stresses can be neglected)

3. Points on a normal to the middle surface before the deformation shall be on a straight line after the deformation has taken place and be normal to the deformed middle surface.

4. Deformations are small compared to the shell thickness.

Consider a shell of small thickness t, in the form of a surface of revolution about the vertical axis. Consider the equilibrium of a small element ds x ds. Suppose P_z is the intensity of loading normal to the surface of the element of membrane in the direction as shown in Fig. 1-1.

Then equating the force acting on an element ds x ds perpendicular to the surface (Z-axis), gives

$$N'_{\phi} \sin\phi/2 \, ds + N'_{\theta} \sin\theta/2 \sin\phi \, ds + P_z \, ds \, ds = 0 \qquad (1.1)$$

For $ds = r_1 d\phi = r_0 d\theta$ and $\sin d\phi/2 = d\phi$, $\sin d\theta/2 = d\theta$ when $d\theta/2$, $d\phi/2$ are very small.

Hence equation (1.1) becomes

$$\frac{N'_{\phi}}{r_1} + \frac{N'_{\theta}}{r_2} + P_z = 0 \qquad (1.2)$$

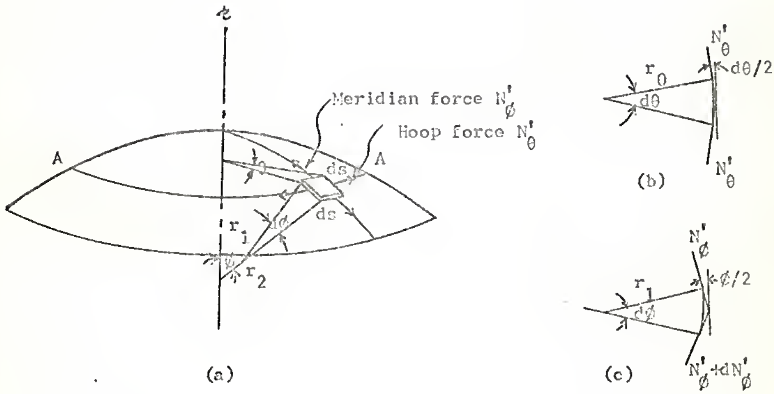

Fig. 1-1 Shell of revolution (dome)

Now consider the vertical equilibrium at section A-A. If R is the total downward loading on the shell above A-A, then

$$- R = N'_{\phi} \, 2\pi r_0 \sin\phi$$

or

$$N'_{\phi} = \frac{- R}{2\pi r_0 \sin\phi} \qquad (1.3)$$

The two equations can be solved for the membrane forces N'_ϕ and N'_θ.

Sign convention adopted is as follows:

For N'_ϕ and N'_θ	Tension	+
	Compression	-
For r_0, r_1	Concave inward	+
	Concave downward	-
For P_z	Acting outward	+
	Acting inward	-

<u>Displacement in symmtrically loaded shells having the form of a surface of revolution</u>

In the case of symmetrical deformation of a shell, a small displacement of a point can be resolved into two components v in the direction of the tangent to the meridian, and w in the direction of the normal to the middle surface (Fig. 1-2). The change in the element due to the difference in radical displacements of the points A and B can be neglected as a small quantity of higher order. Thus the change in length of the element AB due to deformation is

$$\frac{dv}{d\phi}\, d\phi - w\, d\phi$$

Therefore the strain of the shell in the meridional direction is:

$$\epsilon_\phi = \left(\frac{dv}{d\phi}\, d\phi - w\, d\phi\right)/r_1\, d\phi = \frac{dv}{r_1 d\phi} - \frac{w}{r_1} \qquad (1.4)$$

The radius r_0 of the circle increases by the amount

$$v \operatorname{Cos} \phi - w \operatorname{Sin} \phi$$

Hence

$$\epsilon_\theta = \frac{1}{r_0}(v \cos\phi - w \sin\phi) \tag{1.5}$$

Or substituting $r_0 = r_2 \sin\phi$

$$\epsilon_\theta = \frac{v}{r_2} \cot\phi - \frac{w}{r_2} \tag{1.6}$$

Eliminating w from Eqs. (1.4) & (1.6)

$$\frac{dv}{d\phi} - v \cot\phi = r_1 \epsilon_\phi - r_2 \epsilon_\theta \tag{1.7}$$

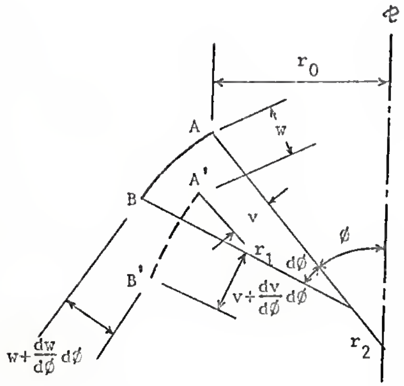

Fig. 1-2 Displacement of the shell

The strain components ϵ_ϕ and ϵ_θ can be expressed in terms of the forces N'_ϕ and N'_θ by applying Hook's law. This gives

$$\begin{aligned} \epsilon_\phi &= \frac{1}{Et}(N'_\phi - \nu N'_\theta) \\ \epsilon_\theta &= \frac{1}{Et}(N'_\theta - \nu N'_\phi) \end{aligned} \tag{1.8}$$

where ν is the poisson's ratio. Substituting in Eq. (1.7)

$$\frac{dv}{d\phi} - v \operatorname{Cot}\phi = \frac{1}{Et}\left[N'_\phi(r_1 + \nu r_2) - N'_\theta(r_2 + \nu r_1) \right] \qquad (1.9)$$

Denoting the right-hand side of this equation by $f(\phi)$, hence yields

$$\frac{dv}{d\phi} - v \operatorname{Cot}\phi = f(\phi)$$

The general solution will be

$$v = \operatorname{Sin}\phi \left[\int \frac{f(\phi)}{\operatorname{Sin}\phi} d\phi + c \right] \qquad (1.10)$$

where c is a constant to be determined by the support conditions. From Eq. (1.6)

$$\begin{aligned} w &= v \operatorname{Cot}\phi - r_2 \epsilon_\theta \\ &= v \operatorname{Cot}\phi - \frac{r_2}{Et}(N'_\theta - \nu N'_\phi) \end{aligned} \qquad (1.11)$$

The meridian rotation $\Delta\phi$ can be expressed in terms of displacement

$$\Delta\phi = \frac{v}{r_1} + \frac{dw}{r_1 d\phi} \qquad (1.12)$$

The horizonal movement ΔH can be derived directly from Eq. (1.8)

$$\Delta H = r_0 \epsilon_\theta = \frac{r_2 \operatorname{Sin}\phi}{Et}(N'_\theta - \nu N'_\phi) \qquad (1.13)$$

The meridian rotation at the edge will be, from Eq. (1.12) with $v = 0$

$$\Delta\phi = \frac{dw}{r_1 d\phi} = \frac{\operatorname{Cot}\phi}{r_1}\frac{dv}{d\phi} - \frac{d}{r_1 d\phi}\left[\frac{r_2}{Et}(N'_\theta - \nu N'_\phi) \right] \qquad (1.14)$$

From Eq. (1.9), with $v = 0$

$$\frac{dv}{d\phi} = \frac{1}{Et}\left[N'_\phi(r_1 + \nu r_2) - N'_\theta(r_2 + \nu r_1) \right] \tag{1.15}$$

Substitute Eq. (1.15) into Eq. (1.14), hence gives

$$\Delta\phi = \frac{\operatorname{Cot}\phi}{r_1 Et}\left[N'_\phi(r_1 + \nu r_2) - N'_\theta(r_2 + \nu r_1) \right] - \frac{d}{r_1 d\phi}\left[\frac{r_2}{Et}(N'_\theta - \nu N'_\phi) \right] \tag{1.16}$$

Where only the horizontal movement is required, it is only necessary to compute N'_ϕ and N'_θ at the edge.

With these Eqs. (1.10), (1.11), (1.12), (1.13), (1.16) the displacement of shells due to membrane theory can be solved.

MEMBRANE AND RIGOROUS ANALYSIS FOR THE PARABOLICAL DOME

The property of the dome

The upper part of the dome is parabolical shape which is shown in the following figure

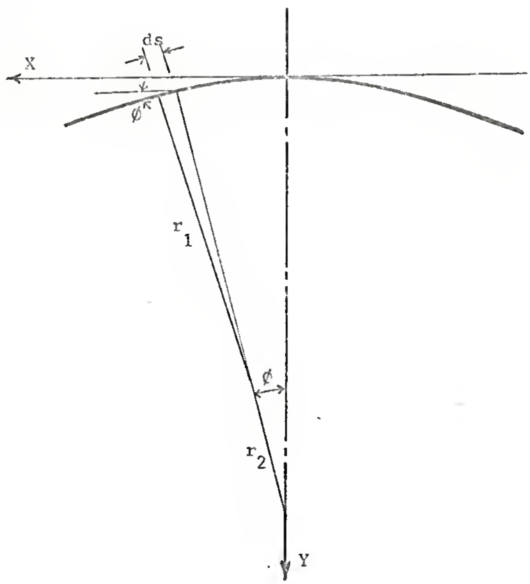

Fig. 2-1 Sect. through the dome

Assume the equation $y = kx^2$ is the function of the curve of the dome, r_1 is the radius of curvature, r_2 is the radius of curvature crossed the axis of revolution of the dome.

$$\frac{dy}{dx} = 2kx = \tan\phi = 2\sqrt{ky} \; ; \qquad \frac{d^2y}{dx^2} = 2k$$

$$\operatorname{Sin}\phi = \frac{dy}{ds} = \frac{dy}{\sqrt{dy^2 + dx^2}} = \frac{dy/dx}{\sqrt{(dy/dx)^2 + 1}}$$

$$= \frac{\operatorname{Tan}\phi}{\sqrt{\operatorname{Tan}^2\phi + 1}} = \frac{2\sqrt{ky}}{\sqrt{4ky + 1}} \qquad (2.1)$$

$$\operatorname{Cos}\phi = \frac{1}{\sqrt{1 + \operatorname{Tan}^2\phi}} = \frac{1}{\sqrt{4ky + 1}} \qquad (2.2)$$

$$r_1 = \frac{\left[1 + (\frac{dy}{dx})^2\right]^{3/2}}{\frac{dy^2}{dx^2}} = \frac{(1 + 4ky)^{3/2}}{2k} = \frac{\operatorname{Sec}^3\phi}{2k} \qquad (2.3)$$

$$r_2 = x\sqrt{1 + (\frac{dx}{dy})^2} = \sqrt{\frac{y}{k}}\sqrt{1 + \frac{1}{4ky}} = \frac{1}{2k}\sqrt{1 + 4ky} = \frac{\operatorname{Sec}\phi}{2k} \qquad (2.4)$$

Assume the thickness of the dome is 6 inches, and the radius of revolution at base is 100'

Let the curve of the dome to be divided into 10 divisions, the property of each point is calculated as shown in table 2.1.

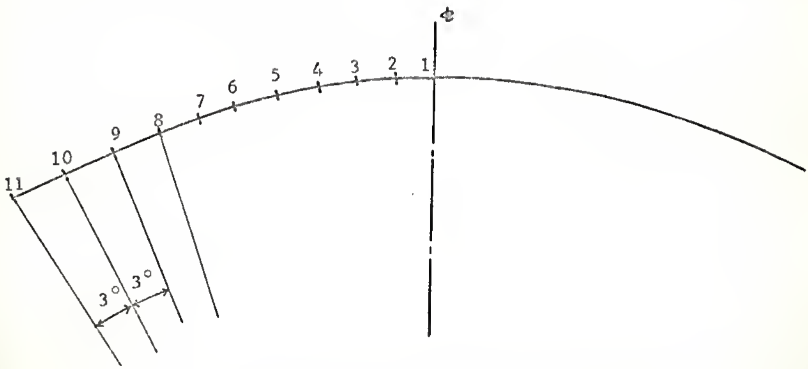

Fig. 2-2 The division of the dome in forces analysis

TABLE 2.1 PROPERTY OF THE DOME

RAD	SINX	COSX	SECX	TANX	DEG
.00000000	0.00000000	1.00000000	1.00000000	0.00000000	0.00
.05235988	.05233596	.99862950	1.00137240	.05240778	3.00
.10471976	.10452847	.99452187	1.00550830	.10510424	6.00
.15707963	.15643446	.98768831	1.01246520	.15838444	9.00
.20943950	.20791168	.97814760	1.02234060	.21255655	12.00
.26179937	.25881903	.96592585	1.03527620	.26794917	15.00
.31415924	.30901697	.95105652	1.05146220	.32491967	18.00
.36651911	.35836792	.93358042	1.07114500	.38386401	21.00
.41887898	.40673661	.91354548	1.09463630	.44522864	24.00
.47123885	.45399046	.89100654	1.12232620	.50952540	27.00
.52359872	.49999996	.86602542	1.15470050	.57735021	30.00

DEG	R1	R2	X	Y
.00	173.20510000	173.20510000	0.00000000	0.00000000
3.00	173.91921000	173.44281000	9.07729530	.23786046
6.00	176.08309000	174.15917000	18.20459200	.95668998
9.00	179.76329000	175.36414000	27.43299500	2.17247990
12.00	185.07489000	177.07461000	36.81588000	3.91272830
15.00	192.18937000	179.31512000	46.41016500	6.21778280
18.00	201.34539000	182.11862000	56.27774400	9.14287300
21.00	212.86560000	185.52778000	66.48720500	12.76102300
24.00	227.18006000	189.59659000	77.11587400	17.16709800
27.00	244.86007000	194.39262000	88.25239500	22.48341700
30.00	266.66667000	200.00002000	100.00000000	28.86751000

Membrane forces due to dead and live load

Let the uniform load acting on the dome as shown in Fig. 2-3

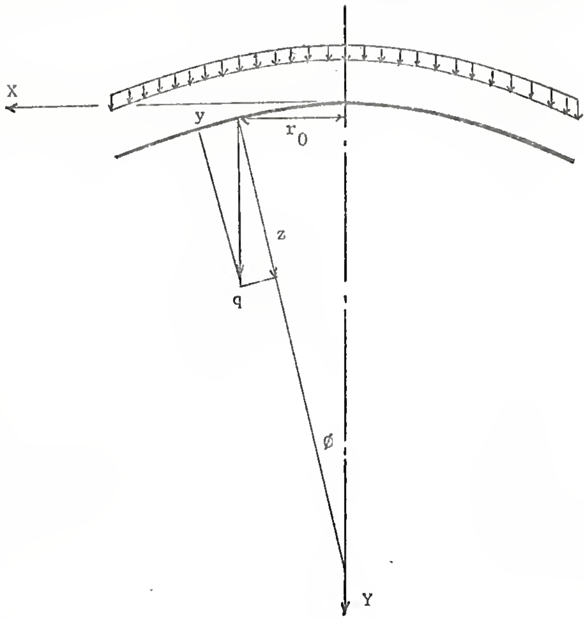

Fig. 2-3 Uniform load acting on the dome

Loading q

Loading in meridional direction $P_\phi = q \sin \phi$

End load at each section is R. S is the length of the curve.

For

$$y = kx^2, \qquad dy = 2kx\,dx,$$

$$ds = \sqrt{dx^2 + dy^2} = \sqrt{1 + 4k^2x^2}\,dx$$

Hence

$$R = \int_{0}^{1+4k^2x^2} 2\pi q x \sqrt{1 + 4k^2x^2}\, dx$$

$$= \frac{\pi q}{6k^2}\left[(1 + 4k^2x^2)^{3/2} - 1\right] \qquad (2.5)$$

The membrane force in meridian direction (Eq. (1.3))

$$N'_{\phi} = \frac{-R}{2\pi r_0 \sin\phi} = \frac{-R\sqrt{1 + 4k^2x^2}}{4\pi k x^2} \qquad (2.6)$$

The membrane force in the direction tangent to the circular cross section can be derived from Eq. (1.2)

$$N'_{\theta} = \frac{R}{2\pi r_1 \sin^2\phi} - \frac{P_z r_0}{\sin\phi}$$

$$= \frac{R}{2\pi r_1 \sin^2\phi} - q x \cot\phi$$

$$= \frac{R}{4y\sqrt{1 + 4ky}} - \frac{q}{2k} \qquad (2.7)$$

For dead load 75 lbs/sq. ft. and live load 30 lbs/sq.ft, use these Eqs. (2.6), (2.5) and (2.7) to calculate the stresses resultants which is given in table 2.2 and 2.3.

TABLE 2.2 MEMBRANE FORCES DUE TO D.L.

DEG	R	N_ϕ	N_θ
.00	0.0000	-6495.1913	-6495.1913
3.00	19427.2990	-6508.4244	-6499.7860
6.00	78301.0750	-6548.9593	-6512.9790
9.00	178424.2800	-6617.1088	-6535.2060
12.00	322936.8000	-6714.6510	-6565.9860
15.00	516500.5800	-6843.5508	-6605.2620
18.00	765609.3000	-7006.6147	-6652.8390
21.00	1079041.2000	-7207.6040	-6708.4340
24.00	1468496.2000	-7451.3656	-6771.7320
27.00	1949518.1000	-7744.1581	-6842.3530
30.00	2542808.0000	-8094.0098	-6919.8750

TABLE 2.3 MEMBRANE FORCES DUE TO L.L.

DEG	R	N_ϕ	N_θ
.00	0.0000	-2598.0765	-2598.0765
3.00	7770.9199	-2603.3698	-2599.9141
6.00	31320.4300	-2619.5837	-2605.1915
9.00	71369.7150	-2646.8436	-2614.0821
12.00	129174.7200	-2685.8604	-2626.3940
15.00	206600.2300	-2737.4203	-2642.1048
18.00	306243.7300	-2802.6459	-2661.1351
21.00	431616.4800	-2883.0415	-2683.3736
24.00	587398.5000	-2980.5465	-2708.6924
27.00	779807.2700	-3097.6634	-2736.9409
30.00	1017123.2000	-3237.6040	-2767.9499

Membrane forces due to snow load

The snow load P acting on the dome is shown in Fig. 2-4.

For

$$P_z = P \cos^2\phi , \qquad P_\phi = P \sin\phi \cos\phi ,$$

$$R = P\pi X^2 \tag{2.8}$$

From Eqs. (1.3) and (1.2), hence

$$N'_\phi = -\frac{R}{2\pi r_0 \sin\phi} = -\frac{P\pi x^2}{2\pi r_2 \sin^2\phi} = -\frac{P x^2}{2 r_2 \sin^2\phi} \tag{2.9}$$

$$N'_\theta = \frac{R}{2\pi r_1 \sin^2\phi} - P_z \frac{r_0}{\sin\phi} = \frac{R}{2\pi r_1 \sin^2\phi} - r_2 P \cos^2\phi \tag{2.10}$$

For P = 30 lbs/sq.ft, the solution of the above Eqs. is shown in table 2.4.

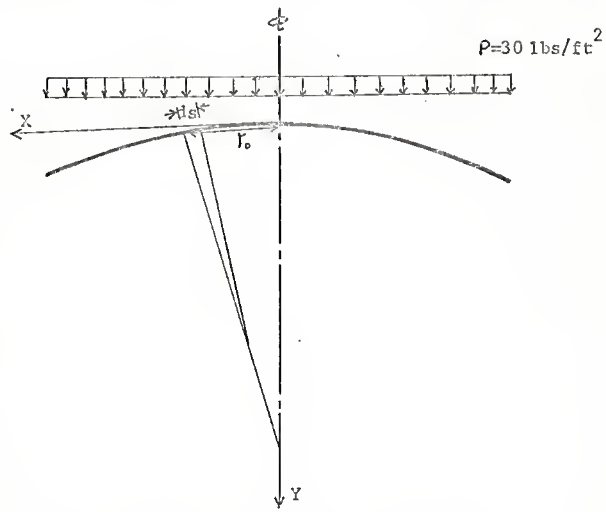

Fig. 2-4 Snow load acting on the dome

TABLE 2.4 MEMBRANE FORCES DUE TO SNOW LOAD

DEG	R	N_ϕ	N_θ
.00	0.0000	-2598.0765	-2598.0765
3.00	7765.7617	-2601.6419	-2594.5165
6.00	31234.3900	-2612.3876	-2583.8437
9.00	70927.9780	-2630.4622	-2566.0902
12.00	127744.2900	-2656.1191	-2541.3022
15.00	203000.6200	-2689.7268	-2509.5495
18.00	298500.1100	-2731.7794	-2470.9174
21.00	416626.8000	-2762.9168	-2425.5135
24.00	560478.1700	-2843.9489	-2373.4613
27.00	734047.4500	-2915.6893	-2314.9030
30.00	942477.8100	-3000.0[illegible]3	-2250.0002

Displacement from the membrane theory

From Eq. (1.13) the horizontal displacement

$$\Delta_H = \frac{r_2 \sin\phi}{Et}(N'_\theta - \nu N'_\phi) \tag{2.11}$$

And from Eq. (1.16), the angle rotation in meridian direction

$$\Delta\phi = \frac{\cot\phi}{r_1 Et}\left[N'_\phi(r_1 + \nu r_2) - N'_\theta(r_2 + \nu r_1)\right] - \frac{d}{r_1 d\phi}\left[\frac{r_2}{Et}(N'_\theta - \nu N'_\theta)\right] \tag{2.12}$$

For

$$\frac{d}{r_1 d\phi}\left(\frac{\Delta H}{\sin\phi}\right) = \frac{1}{r_1}\,\frac{q}{12k^2 Et}\,\frac{d}{d\phi}\Big[(\sec\phi\csc^2\phi - \csc^2\phi\cos^2\phi - 3\sec\phi$$

$$- \nu(\sec^3\phi\csc\phi - \sec\phi\csc\phi\cot\phi)\Big]$$

$$= \frac{q}{12 r_1 k^2 Et}\Big[(-2\csc^3\phi + \csc\phi\sec^2\phi + 2\cos\phi\csc\phi + 2\cot^3\phi$$

$$- 3\sec\phi\tan\phi) - \nu(-\sec^2\phi\csc\phi\cot\phi + 3\sec^4\phi$$

$$+ \csc^3\phi\sec\phi - \csc\phi\sec\phi + \csc^2\phi\cot\phi)\Big]$$

Let

$$\frac{d}{r_1 d\phi}\left(\frac{\Delta H}{\sin\phi}\right) = A$$

Hence

$$\Delta\phi = \frac{\cot\phi}{r_1 Et}\left[N'_\phi(r_1 + \nu r_2) - N'_\theta(r_2 + \nu r_1)\right] - A \tag{2.13}$$

The sign convention see Fig. 2-5. For the numerical calculation of Eq. (2.11) and (2.13) see table 2.5 and 2.6.

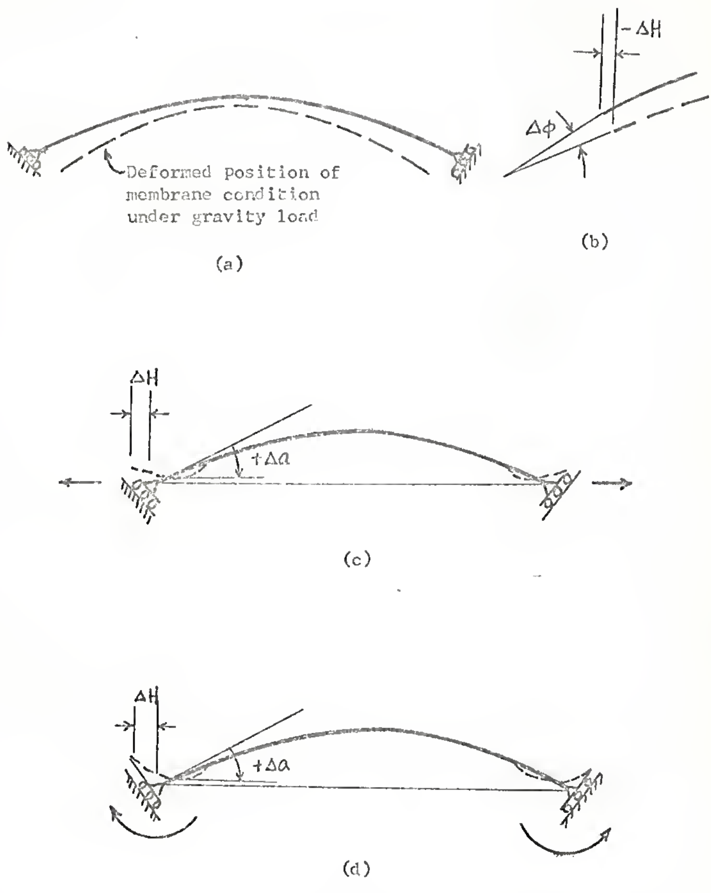

Fig. 2-5 Sign convention of the deformation correspond to the edge forces

TABLE 2.5 DISPLACEMENT FROM MEMBRANE THEORY (U.L.)

DEG	E·ΔH	E·ΔΦ
.00	0.000000	0.000000
3.00	-94369.405000	2.615126
6.00	-189443.800000	-9.816866
9.00	-285949.700000	-15.571705
12.00	-384582.790000	-20.425761
15.00	-486[illegible]58.470000	-24.665059
18.00	-591086.970000	-28.270687
21.00	-700364.680000	-31.191613
24.00	-814568.630000	-33.363228
27.00	-934331.880000	-34.821466
30.00	-1060214.600000	-35.504818

TABLE 2.6 DISPLACEMENT FROM MEMBRANE THEORY (L.L.)

DEG	E·ΔH	E·ΔΦ
.00	0.000000	0.000000
3.00	-37747.758000	5.854326
6.00	-75777.517000	-3.331127
9.00	-114379.860000	-6.053862
12.00	-153833.090000	-8.096722
15.00	-194423.390000	-9.827795
18.00	-236434.740000	-11.265249
21.00	-280145.870000	-12.460994
24.00	-325827.390000	-13.341567
27.00	-373732.710000	-13.919031
30.00	-424[illegible]85.820000	-14.193610

Differential equations of bending in the dome

For the loads symmetrical about the axis of rotation, the formula was derived by Timoshenko

$$\frac{r_2}{r_1^2}\frac{d^2U}{d\phi^2} + \frac{1}{r_1}\left[\frac{d}{d\phi}\left(\frac{r_2}{r_1}\right) + \frac{r_2}{r_1}\operatorname{Cot}\phi - \frac{r_2}{r_1 t}\frac{dt}{d\phi}\right]\frac{dU}{d\phi}$$

$$- \frac{1}{r_1}\left[\frac{r_1}{r_2}\operatorname{Cot}^2\phi - \nu - \frac{\nu}{h}\frac{dh}{d\phi}\operatorname{Cot}\phi\right]U + \frac{r_2}{r_1}\frac{dN'_\phi}{d\phi}$$

$$+ \frac{1}{r_1}\left[\nu\frac{dr_2}{d\phi} + \nu r_2\operatorname{Cot}\phi + r_1\operatorname{Cot}\phi - \frac{\nu r_2}{t}\frac{dt}{d\phi}\right]N'_\phi$$

$$- \frac{r_2}{r_1}\frac{dN'_\theta}{d\phi} - \frac{1}{r_1}\left[\frac{dr_2}{d\phi} + r_2\operatorname{Cot}\phi + \nu r_1\operatorname{Cot}\phi - \frac{r_2}{t}\frac{dt}{d\phi}\right]N'_\theta = EtV \qquad (2.14)$$

$$\frac{r_2}{r_1^2}\frac{d^2v}{d\phi^2} + \frac{1}{r_1}\left[\frac{d}{d\phi}\left(\frac{r_2}{r_1}\right) + \frac{r_2}{r_1}\operatorname{Cot}\phi + 3\frac{r_2}{r_1 t}\frac{dt}{d\phi}\right]\frac{dv}{d\phi}$$

$$- \frac{1}{r_1}\left(\nu - \frac{3\nu\operatorname{Cot}\phi}{t}\frac{dt}{d\phi} + \frac{r_1}{r_2}\operatorname{Cot}^2\phi\right)V = -\frac{U}{D} \qquad (2.15)$$

Where

V	=	ϕy	Angle of rotation in meridian direction
U	=	$r_2 Q_\phi$	Q_ϕ is shearing force
D	=	$\dfrac{Et^3}{(2U - \nu^2)}$	The flexural rigidity for the shell

Because of constant thickness, $\frac{dt}{d\phi} = 0$, $\frac{\nu}{r_1}$ is small when compared with another term, therefore cancel it in calculation. Also neglect the effect of the membrane stress resultants on bending. Then all the term N'_ϕ and N'_θ

are dropped from the equation

$$\frac{d^2U}{d\phi^2} + \frac{r_1}{r_2}\left[\frac{d}{d\phi}\frac{r_2}{r_1} + \frac{r_2}{r_1}\operatorname{Cot}\phi\right]\frac{dU}{d\phi} - \frac{r_1}{r_2}\left[\frac{r_1}{r_2}\operatorname{Cot}^2\phi\right]U = EtV\frac{r_1^2}{r_2} \qquad (2.16)$$

$$\frac{d^2V}{d\phi^2} + \frac{r_1}{r_2}\left[\frac{d}{d\phi}\left(\frac{r_2}{r_1}\right) + \frac{r_2}{r_1}\operatorname{Cot}\phi\right]\frac{dv}{d\phi} - \frac{r_1}{r_2}\left[\frac{r_1}{r_2}\operatorname{Cot}^2\phi\right]V = -\frac{U r_1^2}{D\, r_2} \qquad (2.17)$$

For

$$\frac{r_2}{r_1} = \frac{\operatorname{Sec}\phi\cdot 2k}{\operatorname{Sec}^3\phi\cdot 2k} = \frac{1}{\operatorname{Sec}^2\phi} = \operatorname{Cos}^2\phi$$

$$\frac{d}{d\phi}\frac{r_2}{r_1} = -2\operatorname{Cos}\phi\operatorname{Sin}\phi = -\operatorname{Sin}2\phi$$

Hence

$$\frac{d^2u}{d\phi^2} + \frac{1}{\operatorname{Cos}^2\phi}\left[-\operatorname{Sin}2\phi + \operatorname{Cos}^2\phi\operatorname{Cot}\phi\right]\frac{du}{d\phi} - \frac{1}{\operatorname{Cos}^2\phi}\left[\frac{1}{\operatorname{Cos}^2\phi}\operatorname{Cot}^2\phi\right]U$$

$$= EtV\operatorname{Sec}^5\phi/2k \qquad (2.18)$$

$$\frac{d^2V}{d\phi^2} + \frac{1}{\operatorname{Cos}^2\phi}\left[-\operatorname{Sin}2\phi + \operatorname{Cos}^2\phi\operatorname{Cot}\phi\right]\frac{dV}{d\phi} - \frac{1}{\operatorname{Cos}^2\phi}\left[\frac{1}{\operatorname{Cos}^2\phi}\operatorname{Cot}^2\phi\right]V$$

$$= -\frac{U}{D}\frac{\operatorname{Sec}^5\phi}{2k} \qquad (2.19)$$

To simplify the above equation, then get

$$\frac{d^2U}{d\phi^2} + \left[-2\operatorname{Tan}\phi + \operatorname{Cot}\phi\right]\frac{dU}{d\phi} - \frac{4}{\operatorname{Sin}^2 2\phi}U = EtV\operatorname{Sec}^5\phi/2k \qquad (2.20)$$

$$\frac{d^2V}{d\phi^2} + \left[-2\operatorname{Tan}\phi + \operatorname{Cot}\phi\right]\frac{d}{d\phi} - \frac{4}{\operatorname{Sin}^2 2\phi}V = -\frac{U}{D}\frac{\operatorname{Sec}^5\phi}{2k} \qquad (2.21)$$

The integration of these two equations is difficult. Finite differences can be applied to solution of those problems.

Application of finite difference equations in bending analysis

The application of finite difference equations to the solution of difficult structural problems is in large measure comparable to the technique now used to surmount mathematical difficulties in the solution of complicated differential equations. Essentially, the technique employed consists of replacing the derivatives of differential equation by its central difference equivalent. The problem is thus reduced to the simple task of solving a system of simultaneous linear algebraic equations.

Just as the replacement of an integral by the summation procedure involves the use of average value of the ordinate, so the replacement of a derivative by finite differences is based on taking the difference of average value of the ordinate. In this light then, it is evident from geometrical considerations of Fig. 2-6. That if $y = f(x)$, then

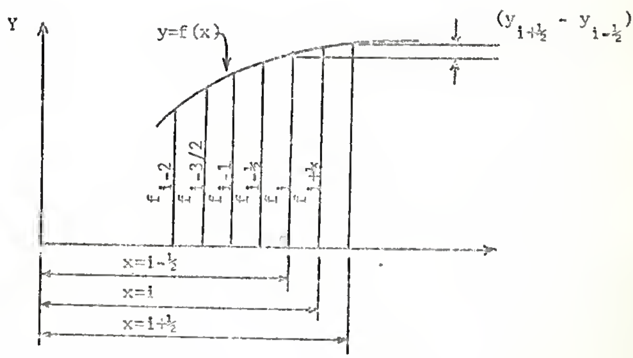

Fig. 6

Let the distance between each point in the x ordinate is h, then

$$\frac{dy}{dx_i} \doteqdot \frac{y_{i+\frac{1}{2}} - y_{i-\frac{1}{2}}}{h}$$

In which $y_{i+\frac{1}{2}}$ represents the ordinate at $x = i+\frac{1}{2}$ and $\doteqdot$ means approximately equal to. By repeating this process, it naturally follows that

$$\frac{d^2y}{dx_i^2} \doteqdot \frac{d}{dx}\left(\frac{dy}{dx}\right)_i \doteqdot \frac{y_{i+1} - y_i}{h^2} - \frac{y_i - y_{i-1}}{h^2} = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

$$\frac{d^3y}{dx_i^3} \doteqdot \frac{y_{i+\frac{3}{2}} - 3y_{i+\frac{1}{2}} + 3y_{i-\frac{1}{2}} - y_{i-}}{h^3}$$

$$\frac{d^4y}{dx_i^4} \doteqdot \frac{y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2}}{h^4}$$

.

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From these expression of derivative, the finite difference equation of equations (2.20) and (2.21) will therefore be

$$\frac{U_{i+1} - 2U_i + U_{i-1}}{(\Delta\phi)^2} + (-2\operatorname{Tan}\phi_i + \operatorname{Cot}\phi_i)\frac{U_{i+1} - U_{i-1}}{2\Delta\phi} - \frac{4}{\operatorname{Sin}^2(2\phi_i)}U_i$$

$$= EhV_i \operatorname{Sec}^5\phi_i/2k$$

$$\frac{V_{i+1} - 2V_i + V_{i-1}}{(\Delta\phi)^2} + (-2\operatorname{Tan}\phi_i + \operatorname{Cot}\phi_i)\frac{V_{i+1} - V_{i-1}}{2\ \phi} - \frac{4}{\operatorname{Sin}^2(2\phi_i)}$$

$$= -\frac{U_i}{D}\frac{\operatorname{Sec}^5\phi_i}{2k}$$

Collecting similar terms of U's and V's and divide both side by E

$$\frac{U_{i+1}}{E}\left[1 + (-2\,\mathrm{Tan}\,\phi_i + \mathrm{Cot}\,\phi_i)\Delta\phi/2\right] - \frac{U_i}{E}\left[2 + 4\Delta\phi^2/\mathrm{Sin}^2(2\phi_i)\right]$$

$$+ \frac{U_{i-1}}{E}\left[1 + (2\,\mathrm{Tan}\,\phi_i - \mathrm{Cot}\,\phi_i)\Delta\phi/2\right] - tV_i\Delta\phi^2\,\mathrm{Sec}^5\phi_i/2k = 0 \qquad (2.22)$$

$$V_{i+1}\left[1 + (-2\,\mathrm{Tan}\,\phi_i + \mathrm{Cot}\,\phi_i)\Delta\phi/2\right] - V_i\left[2 + 4\Delta\phi^2/\mathrm{Sin}^2 2\phi_i\right]$$

$$+ V_{i-1}\left[1 + 2\,\mathrm{Tan}\,\phi_i - \mathrm{Cot}\,\phi_i)\Delta\phi/2\right] + \frac{6(1-\nu^2)\Delta\phi^2\,\mathrm{Sec}^5\phi_i}{Et^3k} = 0 \qquad (2.23)$$

Since the trigonometric functions for the various values of ϕ can be readily evaluated, two difference equations for each point can be written. Because of symmetry, U and V are zero at $\phi = 0$. It follows that the equations at $\Delta\phi = 0$ are superfluous. At boundary, it is assumed $\Delta H = 1$, $\Delta\phi = 0$; $\Delta H = 0$, $\Delta\phi = 1$ in order to get the expression of stiffness both for displacement and rotation of the dome shell.

Since

$$N_\phi = -\frac{1}{r_2}\,U\,\mathrm{Cot}\,\phi \qquad (2.24)$$

$$N_\theta = -\frac{1}{r_1}\frac{dU}{d\phi} \qquad (2.25)$$

$$E\Delta H = \frac{r_2\,\mathrm{Sin}\,\phi}{t}(N_\theta - \nu N_\phi) \qquad (2.26)$$

Substitute (2.24) and (2.25) into Eq. (2.26)

i.e.
$$\Delta H = \frac{r_2 \operatorname{Sin}\phi}{Et}\left(-\frac{1}{r_1}\frac{U_{i+1} - U_{i-1}}{2\ \phi} + \frac{\nu U_i}{r_2}\operatorname{Cot}\phi\right)$$

$$= -\frac{r_2 \operatorname{Sin}\phi}{2r_1 \Delta\phi\, tE}U_{i+1} + \frac{\nu \operatorname{Cos}\phi}{tE}U_i + \frac{r_2 \operatorname{Sin}\phi}{2\Delta\phi\, tr_1 E}U_{i-1} \tag{2.27}$$

For $\Delta H = 1$, $\Delta\phi = 0$, hence

$$-\frac{r_2 \operatorname{Sin}\phi}{E\cdot 2r_1 \Delta\phi\, t}U_{i+1} + \frac{\nu \operatorname{Cos}\phi}{Et}U_i + \frac{r_2 \operatorname{Sin}\phi}{E\cdot 2\Delta\phi\, tr_1}U_{i-1} = 1 \tag{2.28}$$

$$V_i = 0 \tag{2.29}$$

Now there are 10 points, and therefore 22 simultaneous equations for total 22 unknowns are obtained. Let the coefficient of Eqs. (2.22) and (2.28) to be C_i^n and D_i^n for Eqs. (2.23) and (2.29), then writing in matrix form, it is:

$$\begin{bmatrix} C_2^1 & C_2^2 & C_2^3 \\ D_2^1 & D_2^2 & D_2^3 \\ C_3^1 & C_3^2 & C_3^3 \\ \vdots & \vdots & \vdots \\ C_{11}^1 & C_{11}^2 & C_{11}^3 \\ D_{11}^1 & D_{11}^2 & D_{11}^3 \end{bmatrix} \begin{Bmatrix} U_2 \\ V_2 \\ U_3 \\ \vdots \\ U_{11} \\ V_{11} \end{Bmatrix} = \begin{Bmatrix} C_2^4 \\ D_2^4 \\ C_3^4 \\ \vdots \\ C_{11}^4 \\ D_{11}^4 \end{Bmatrix} \tag{2.30}$$

i.e.

$$(A_{ij})(U_i) = (G_i) \tag{2.31}$$

Hence

$$(U_i) = (A_{ij})^{-1}(G_i) \tag{2.32}$$

Use the boundary condition $\Delta H = 0$, $\Delta\phi = 1$ so U's and V's for the unit rotation at edge of the dome shell can be got.

Then substitute these U's and V's into following equation, the force due to these boundary displacement and rotation can be obtained

$$N_\phi = -\frac{1}{r_2} U_i \operatorname{Cot}\phi \tag{2.33}$$

$$N_\theta = -\frac{1}{r_1}\frac{dU_i}{d\phi} = -\frac{(U_{i+1} - U_{i-1})}{r_1 \cdot 2\Delta\phi} \tag{2.34}$$

$$M_\theta = -D(V_i \frac{\operatorname{Cot}\phi_i}{r_2} + \frac{\nu\, dV_i}{r_1 d\phi_i}) = -D(V_i \frac{\operatorname{Cot}\phi_i}{r_2} + \frac{\nu}{2r_1\Delta\phi}(V_{i+1} - V_{i-1})) \tag{2.35}$$

$$M_\phi = -D(\frac{1}{r_1}\frac{dV_i}{d\phi} + \nu\frac{V_i \operatorname{Cot}\phi}{r_2}) = -D[\frac{1}{2r_1\Delta\phi}(V_{i+1} - V_{i-1}) + \nu\frac{V_i \operatorname{Cot}\phi}{r_2}] \tag{2.36}$$

The matrix A, A^{-1}, U, V and the solution of Eqs. (2.33) to (2.36) are listed in table 2.7 to 2.13.

TABLE 2.7 COEFFECIENT MATRIX OF AX=G

-3.00366330	-.23905952	1.49679890	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	43.03071300	-3.00366330
.00000000	1.49679890	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.75641780	0.00000000	-2.25368760	-.24403729
1.24358220	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	.75641780
43.92671300	-2.25368760	0.00000000	1.24358220
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	.84299940	0.00000000
-2.11483980	-.25259716	1.15700060	0.00000000
.00000000	0.00000000	0.00000000	0.00000000

.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	.84299940	45.46748800	-2.11463980
.00000000	1.15700060	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.88796250	0.00000000	-2.06628730	-.26515877
1.11203750	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	.88796250
47.72857800	-2.06628730	0.00000000	1.11203750
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	.91632500	0.00000000
-2.04386490	-.28236385	1.08367500	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000

.00000000	.91632500	50.82549300	-2.04386490
.00000000	1.08367500	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.93643920	0.00000000	-2.03174090	-.30513799
1.06356080	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	.93643920
54.92483600	-2.03174090	0.00000000	1.06356080
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	.95189800	0.00000000
-2.02449260	-.33478751	1.04810200	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	.95189800	60.26175300	-2.02449260
.00000000	1.04810200	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000

.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.96451110	0.00000000	-2.01985680	-.37314458
1.03548890	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	.96451110
67.16602500	-2.01985680	0.00000000	1.03548890
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	.97529770	0.00000000
-2.01675490	-.42278880	1.02470230	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	.97529770	76.10198400	-2.01675490
.00000000	1.02470230	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000

.98488500	0.00000000	-2.01462160	-.48738785
1.01511500	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	.98488500
87.72981600	-2.01462160	0.00000000	1.01511500
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
7.16197200	0.00000000	.80000006	0.00000000
-7.16197200	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000	0.00000000
.00000000	1.00000000	0.00000000	0.00000000

TABLE 2.8 INVERSION OF THE MATRIX A

- .14273265	0.12810095E-01	0.26508793E-01
.57582041E-02	0.24253473E-01	0.32288604E-03
.41055047E-02	-0.39594981E-03	-0.74210921E-03
- .11253090E-03	-0.38743407E-03	0.82051808E-06
- .29443566E-04	0.56656773E-05	0.11433501E-04
.76734157E-06	0.24757670E-05	-0.12502910E-06
.13343578E-05	0.00000000	0.18912760E-06
.77846737E-06		
- .23058168E+01	-0.14273266	-0.10364767E+01
.26508793E-01	-0.58119515E-01	0.24253475E-01
.71270961E-01	0.41055050E-02	0.20255561E-01
- .74210950E-03	-0.14765269E-03	-0.38743258E-03
- .10198211E-02	-0.29427080E-04	-0.13919557E-03
.11466737E-04	0.17990982E-04	0.21159211E-05
.96965543E-05	0.00000000	0.13743585E-05
.25577934E-05		
.13396404E-01	0.29099485E-02	-0.11234381
.15788950E-01	0.39387568E-01	0.45215591E-02
.19621577E-01	-0.13885644E-03	0.17458853E-02
- .34434372E-03	-0.80105564E-03	-0.60231805E-04
- .22196470E-03	0.66695421E-05	0.71242195E-06
.33712399E-05	0.78415920E-05	0.87042940E-07
.42263629E-05	-0.10000000E-14	0.59903112E-06
.19706849E-05		
- .52379077	0.13396406E-01	-0.28420111E+01

- .11234384	-0.81388073	0.39387562E-01
.24994088E-01	0.19621578E-01	0.61981861E-01
.17458848E-02	0.10841847E-01	-0.80105923E-03
- .12000423E-02	-0.22193174E-03	-0.60802311E-03
.95066606E-06	-0.35071651E-04	0.78404691E-05
- .18902479E-04	0.00000000	-0.26791766E-05
- .17246985E-04		
.83085462E-02	0.11061150E-03	0.26700035E-01
.30650741E-02	-0.10308586	0.15727129E-01
.37966875E-01	0.38396830E-02	0.15778539E-01
- .21298304E-03	0.91151841E-03	-0.26685225E-03
- .61984059E-03	-0.34910587E-04	-0.12498027E-03
.58293617E-05	0.58226957E-05	0.17723870E-05
.31382435E-05	0.00000000	0.44480459E-06
- .28662721E-06		
- .19910071E-01	0.83085457E-02	-0.55171334
.26700037E-01	-0.28308833E+01	-0.10308586
- .69114289	0.37966873E-01	0.38336874E-01
.15778534E-01	0.48033461E-01	0.91148701E-03
.62859299E-02	-0.61988383E-03	-0.10423903E-02
- .12433325E-03	-0.35148821E-03	0.98473023E-05
- .18944071E-03	0.00000000	-0.26850718E-04
- .10242165E-03		
.10793902E-02	-0.10410036E-03	0.10208164E-01
- .72240120E-04	0.29138405E-01	0.29468389E-02
- .95788970E-01	0.15408542E-01	0.35938687E-01

.33063683E-02	0.12736485E-01	-0.24488764E-03
.40108994E-03	-0.20400472E-03	-0.45654129E-03
- .18945554E-04	-0.71807483E-04	0.53261312E-05
- .38701898E-04	0.10000000E-13	-0.54854827E-05
- .24320532E-04		
.18738067E-01	0.10793905E-02	0.13003242E-01
.10208167E-01	-0.53043102	0.29138411E-01
- .27735376E+01	-0.95788943E-01	-0.59514647
.35938700E-01	0.44078574E-01	0.12736417E-01
.36722505E-01	0.40054426E-03	0.34491028E-02
- .45703750E-03	-0.84573767E-03	-0.57363860E-04
- .45582509E-03	0.00000000	-0.64607186E-04
- .16338273E-03		
- .16077184E-03	-0.24378895E-04	0.74844485E-03
- .14761688E-03	0.99783324E-02	-0.13468997E-03
.29613675E-01	0.27244663E-02	-0.87730331E-01
.14883030E-01	0.33448211E-01	0.27949766E-02
.99964755E-02	-0.25567959E-03	0.73911638E-04
- .14883555E-03	-0.33973675E-03	-0.51967947E-05
- .18310705E-03	0.00000000	-0.25953006E-04
- .83918983E-04		
.43881998E-02	-0.16077189E-03	0.26571037E-01
.74844437E-03	0.24244224E-01	0.99783302E-02
- .49040382	0.29613683E-01	-0.26789455E+01
- .87730230E-01	-0.50310099	0.33448401E-01
.46000390E-01	0.99951035E-02	0.26835888E-01

.63196184E-04	0.17911634E-02	-0.34304867E-03
.96537880E-03	0.00000000	0.13682969E-03
.82203663E-03		
- .72530467E-04	0.15356393E-06	-0.29674745E-03
- .22312865E-04	0.49812420E-03	-0.14582872E-03
.90690180E-02	-0.17436747E-03	0.28903696E-01
.24152563E-02	-0.78773321E-01	0.14193894E-01
.30500613E-01	0.22946141E-02	0.75178345E-02
- .24822483E-03	-0.11333380E-03	-0.10369028E-03
- .61083230E-04	0.00000000	-0.86577420E-05
.76480432E-04		
- .27649322E-04	-0.72530208E-04	0.40162698E-02
- .29674881E-03	0.26249136E-01	0.49810654E-03
.31387015E-01	0.90689670E-02	-0.43474161
.28903862E-01	-0.25549011E+01	-0.78771531E-01
- .41313765	0.30504211E-01	0.44230829E-01
.74861915E-02	0.20027363E-01	-0.35476659E-03
.10794097E-01	0.00000000	0.15299207E-02
.56244417E-02		
- .49333412E-05	0.94929763E-06	-0.73593007E-04
.22104288E-05	-0.30316540E-03	-0.17080353E-04
.25561159E-03	-0.13001649E-03	0.77313542E-02
- .19765030E-03	0.27298372E-01	0.20542362E-02
- .69065840E-01	0.13360292E-01	0.26986311E-01
.18046624E-02	0.58285220E-02	-0.29528919E-03
.31413838E-02	0.00000000	0.44524965E-03

.18336556E-02		
- .17087364E-03	-0.49305821E-05	-0.39803457E-03
- .73582073E-04	0.30734743E-02	-0.30318665E-03
.23401921E-01	0.25526391E-03	0.35594025E-01
.77302924E-02	-0.36966663	0.27301590E-01
- .24048524E+01	-0.69027022E-01	-0.32737231
.27064308E-01	0.42534443E-01	0.49791423E-02
.22924681E-01	0.00000000	0.32492708E-02
.60710730E-02		
.17629240E-05	0.11923604E-06	0.21736182E-06
.10306326E-05	-0.56252906E-04	0.26065218E-05
- .26774575E-03	-0.11237663E-04	0.52604504E-04
- .10610987E-03	0.61919089E-02	-0.20238787E-03
.24834035E-01	0.16736825E-02	-0.59271708E-01
.12356244E-01	0.24947644E-01	0.11142490E-02
.13445970E-01	0.00000000	0.19057886E-02
.54116284E-02		
- .21296864E-04	0.17680488E-05	-0.18514773E-03
.29005632E-06	-0.47227686E-03	-0.55961674E-04
.19999646E-02	-0.26803665E-03	0.19067401E-01
.44978415E-04	0.36800145E-01	0.61658468E-02
- .29893182	0.24905804E-01	-0.22241236E+01
- .58283200E-01	-0.27114763	0.26917783E-01
- .14613977	0.00000000	-0.20713380E-01
- .98809452E-01		
.35954724E-06	-0.14515384E-07	0.22534720E-05

.55992720E-07	0.24684184E-05	0.82781546E-06
- .39664688E-04	0.25953583E-05	-0.22774386E-03
- .66706283E-05	-0.87919007E-04	-0.86312739E-04
.50518901E-02	-0.20461593E-03	0.23497484E-01
.14188122E-02	-0.54474981E-01	0.12148518E-01
- .29360245E-01	0.10000000E-10	-0.41614268E-02
- .26758441E-01		
.32683589E-05	0.30728813E-06	-0.45025139E-05
.22531497E-05	-0.13524658E-03	0.41745785E-05
- .52956417E-03	-0.31686373E-04	0.62706400E-03
- .22996395E-03	0.14478849E-01	-0.27521141E-03
.46069718E-01	0.43156870E-02	-0.18890632
.25353096E-01	-0.21867333E+01	-0.24405922E-01
- .11785782E+01	0.00000000	-0.16704788
- .54941582		
.37822546E-06	-0.15269448E-07	0.23705381E-05
.58901500E-07	0.25966509E-05	0.87081985E-06
- .41725240E-04	0.27301853E-05	-0.23957497E-03
- .70171626E-05	-0.92486340E-04	-0.90796620E-04
.53143319E-02	-0.21545597E-03	0.24718161E-01
.14925184E-02	-0.57304912E-01	0.12779625E-01
- .55686014	0.00000000	-0.78927569E-01
- .28450217		
.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000

.00000000	0.00000000	0.00000000
.00000000	0.10000000E-10	0.00000000
.00000000	0.00000000	-0.10000000E-08
.00000000	0.00000000	0.00000000
.10000000E+01		
.40179543E-06	-0.16220997E-07	0.25182635E-05
.62572082E-07	0.27584670E-05	0.92508698E-06
- .44325442E-04	0.29003228E-05	-0.25450462E-03
- .74544528E-05	-0.98249830E-04	-0.96454817E-04
.56455068E-02	-0.22888258E-03	0.26258528E-01
.15855280E-02	-0.60876002E-01	0.13576016E-01
- .91562130E-01	0.00000000	-0.15260406
- .58537642E-01		
- .35858605E-04	0.10215024E-05	-0.20050183E-03
- .72765266E-05	-0.93192791E-04	-0.79309581E-04
.41198361E-02	-0.20520948E-03	0.20096524E-01
.82956360E-03	-0.60546759E-02	0.81139797E-02
- .50398106	0.14433298E-01	-0.19529499E+01
- .15358679	0.70741057E+01	-0.10807812E+01
.49269298E+02	0.98511006	0.69832721E+01
.27105358E+02		

TABLE 2.9 E.U AND E.V VECTOR, FOR $\Delta H=1$, $\Delta\phi=0$

DEG	E.U	E.V
.00	.00000018912760	.00000137435850
3.00	.00000059903112	-.00000267917660
6.00	.00000044480459	-.00002685071800
9.00	-.00000548548270	-.00006460718600
12.00	-.00002595300600	.00013682969000
15.00	-.00000865774200	.00152992070000
18.00	.00044524965000	.00324927080000
21.00	.00190578860000	-.02071338000000
24.00	-.00416142680000	-.16704788000000
27.00	-.07892756900000	0.00000000000000
30.00	-.15260406000000	6.98327210000000

TABLE 2.10 E.U AND E.V VECTOR, FOR $\Delta H=0$ $\Delta\phi=1$

DEG	E.U	E.V
.00	.00000077846737	.00000255779340
3.00	.00000197068490	-.00001724698500
6.00	-.00000028662721	-.00010242165000
9.00	-.00002432053200	-.00016338273000
12.00	-.00008391898300	.00082203663000
15.00	.00007648043200	.00562444170000
18.00	.00183365560000	.00607107300000
21.00	.00541162840000	-.09880945200000
24.00	-.02675844100000	-.54941582000000
27.00	-.28450217000000	1.00000000000000
30.00	-.05853764200000	27.10535800000000

TABLE 2.11 FOCES DUE TO BOUNDARY DISP. FOR $\Delta H=1$

DEG	N_ϕ	N_θ
.00	0.00000000	0.00000000
3.00	-.00000002	-.00000003
6.00	-.00000003	-.00000001
9.00	-.00000002	.00000032
12.00	.00000015	.00000136
15.00	.00000054	.00000016
18.00	.00000015	-.00002235
21.00	-.00000625	-.00008588
24.00	-.00002258	.00019364
27.00	.00004201	.00315242
30.00	.00068353	.00531571

TABLE 2.12 FOCES DUE TO BOUNDARY DISP. FOR $\Delta\phi=1$

DEG	N_ϕ	N_θ
.00	0.00000000	0.00000000
3.00	-.00000009	-.00000011
6.00	-.00000011	.00000006
9.00	.00000001	.00000140
12.00	.00000065	.00000432
15.00	.00000175	-.00000501
18.00	-.00000129	-.00009095
21.00	-.00002575	-.00023934
24.00	-.00006411	.00120184
27.00	.00027016	.01130635
30.00	.00246386	.00113801

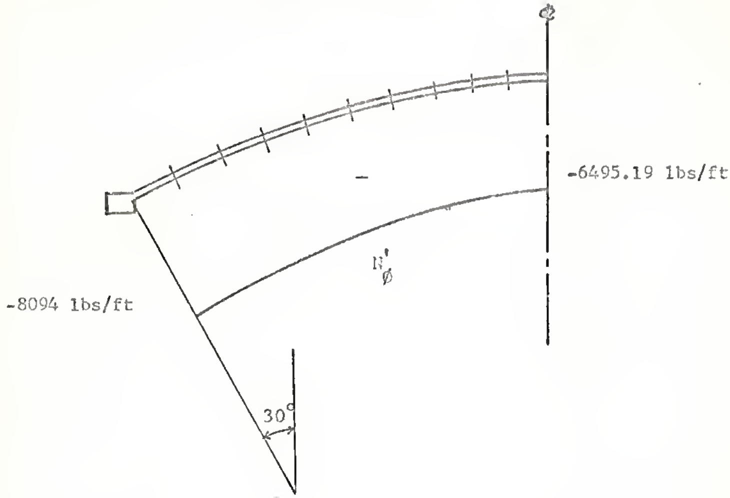
TABLE 2.13 MOMENT DUE TO BOUNDARY DISP. FOR $\Delta H=1$

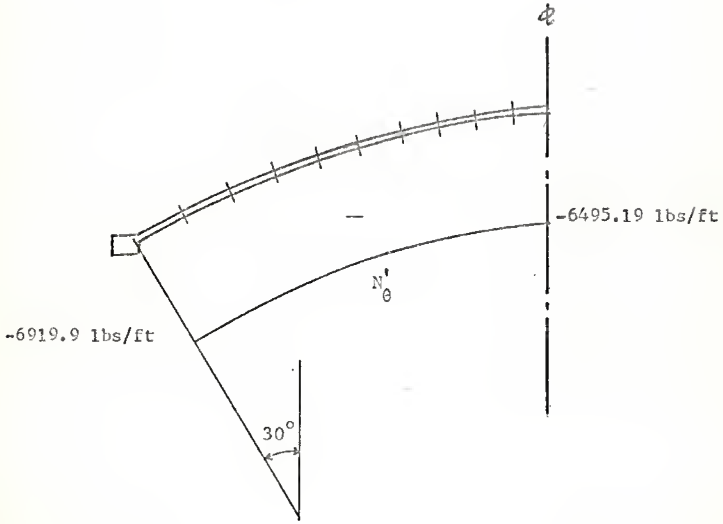
DEG	M_ϕ	M_θ
.00	0.00000000	0.00000000
3.00	.00000000	-.00000000
6.00	.00000002	.00000000
9.00	.00000004	.00000002
12.00	-.00000009	.00000000
15.00	-.00000087	-.00000020
18.00	-.00000166	-.00000060
21.00	.00001073	.00000167
24.00	.00007820	.00001820
27.00	-.00000511	.00001655
30.00	-.00277834	-.00055567

TABLE 2.14 MOMENT DUE TO BOUNDARY DISP. FOR $\Delta\phi=1$

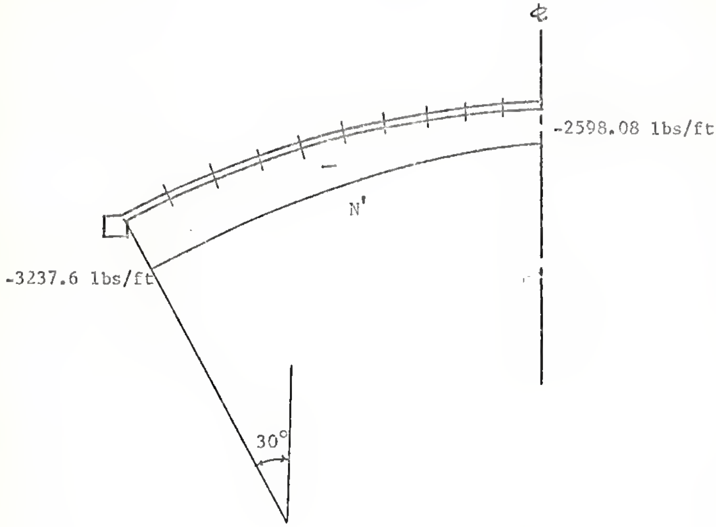
DEG	M_ϕ	M_θ
.00	0.00000000	0.00000000
3.00	.00000000	-.00000000
6.00	.00000006	.00000002
9.00	.00000009	.00000006
12.00	-.00000051	-.00000006
15.00	-.00000316	-.00000081
18.00	-.00000291	-.00000157
21.00	.00005065	.00000924
24.00	.00025590	.00006337
27.00	-.00045294	-.00003281
30.00	-.01076437	-.00224308

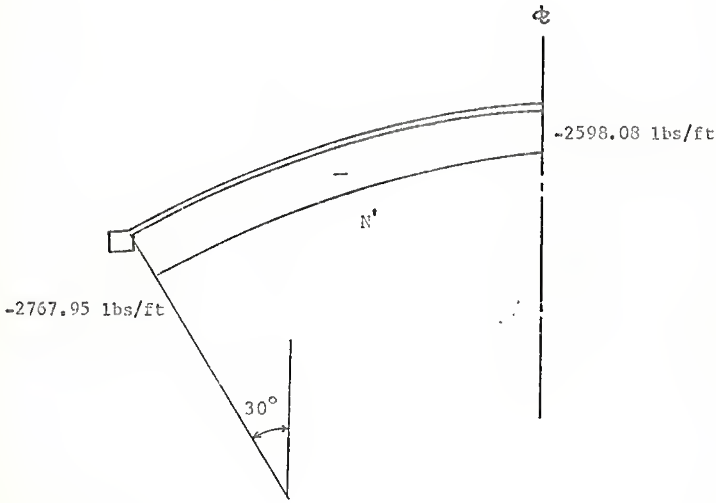
Diagram of the forces



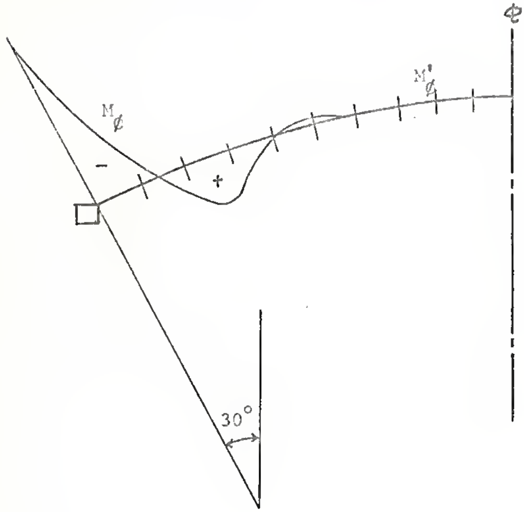


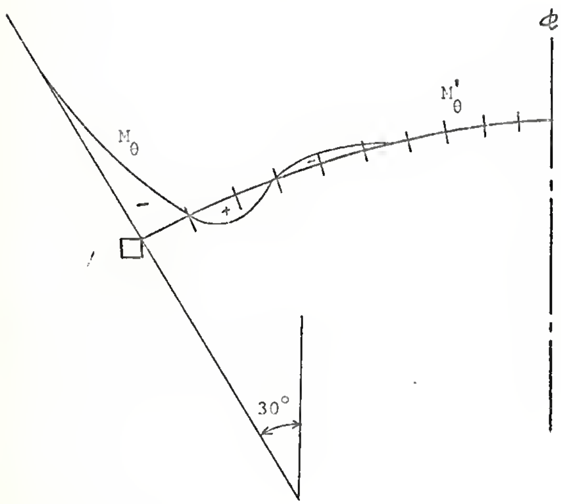
Dia. 2-1 Dead load membrane force



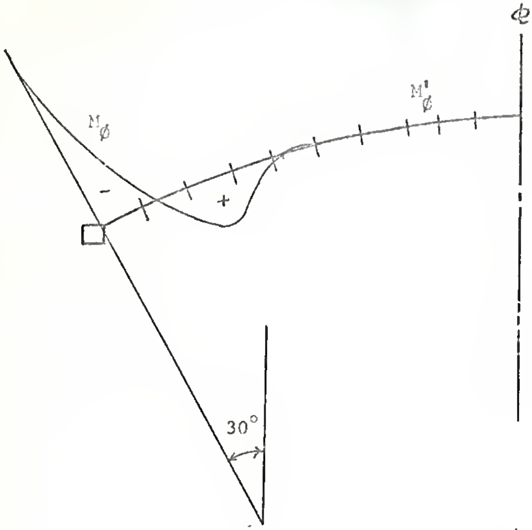


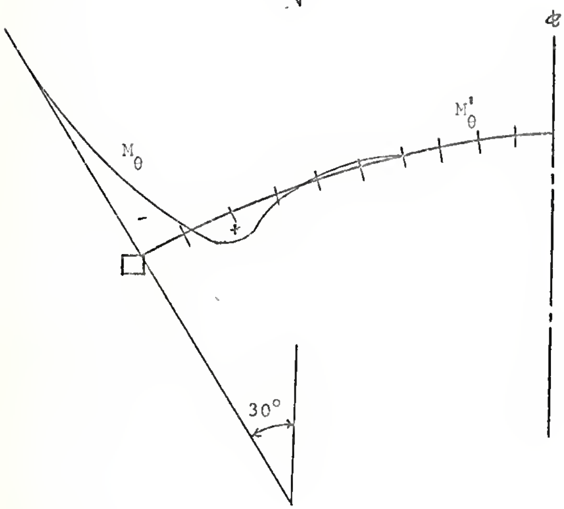
Dia. 2-2 Live load membrane force



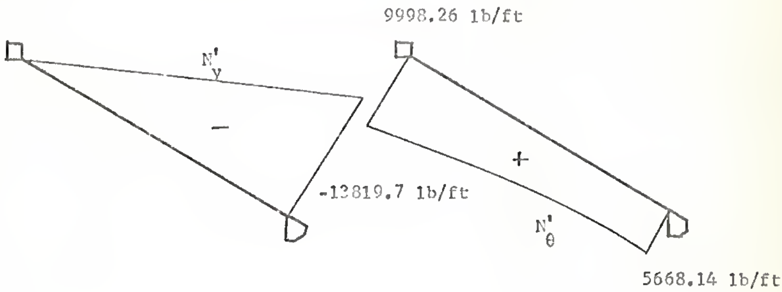


Dia. 2-3 Bending moment due to $\Delta H = 1$ at boundary

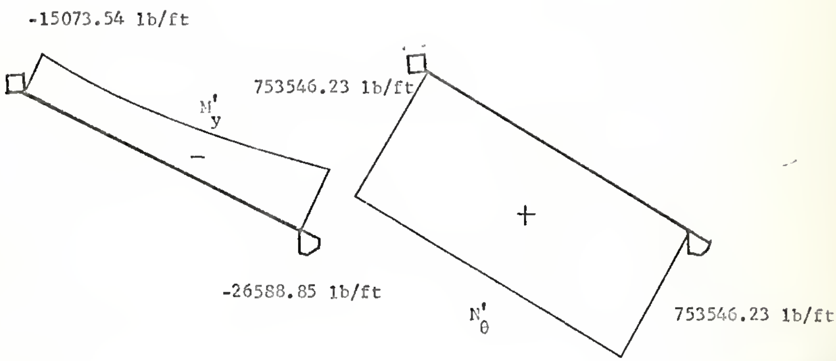




Dia. 2-4 Bending moment due to $\phi = 1$ at boundary



Dia. 3-1 Membrane forces due D.L.



Dia. 3-2 Membrane forces due to Dome L. P

MEMBRANE AND RIGOROUS ANALYSIS FOR CONICAL SHELL WALL

Membrane forces due to uniform distributed load and dome load

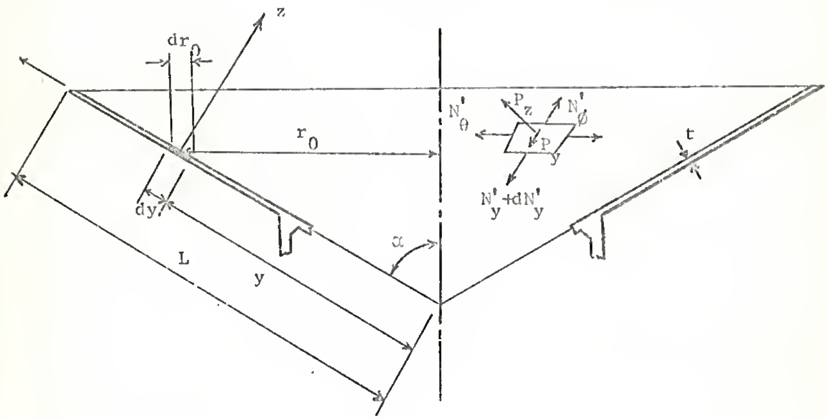


Fig. 3-1 Section through the conical shell wall

From the figure shown above $r_0 = y \sin\alpha$, the shell is loaded with uniformed load q. Then the normal forces $P_z = -q \sin\alpha$, and tangential forces $P_y = -q \cos\alpha$ per unit area, distributed uniformly with respect to the axis of the cone. The static equilibrium will require that along any hoop circle at a distance y from the apex, it will be

$$\frac{d}{dy}(N'_y \cos\alpha)\, y \sin\alpha\, d\phi = -y\,(P_y \sin\alpha\, d\theta \cos\alpha + P_z \sin^2\alpha\, d\theta)$$

Which gives

$$N'_y = -\int_y^L (P_z \tan\alpha + P_y)\, y\, dy \tag{3.1}$$

And

$$N'_y = \frac{q}{y}\left(\frac{\sin^2\alpha}{\cos\alpha} + \cos\alpha\right)\int_y^L y\,dy = \frac{q\,y^2}{2y\cos\alpha} + \frac{C}{y} \qquad (3.2)$$

Assume the base of the cone is fixed, $N'_y = 0$ at $y = L$. Hence

$$C = -\frac{q\,L^2}{2\cos\alpha}$$

$$N'_y = -\frac{q}{2}\,\frac{L^2 - y^2}{y\cos\alpha} \qquad (3.3)$$

The hoop force N'_θ can be derived from the equilibrium condition in a direction perpendicular to the surface, which gives directly

$$N'_\theta = -P_z\,y\tan\alpha = q\,y\sin\alpha\tan\alpha \qquad (3.4)$$

For concentrated dome load P lbs per liner foot acting along the top edge, the static vertical equilibrium equation is

$$N'_y \cdot 2\pi y\sin\alpha\cos\alpha + R = 0$$

$$N'_y = \frac{-R}{2\pi y\sin\alpha\cos\alpha} \qquad (3.5)$$

Where R is total loading due to P

Using the equilibrium condition in a direction perpendicular to the surface, yields

$$N'_\theta = \frac{R\tan\alpha}{2\pi y\sin\alpha}\cdot y\sin\alpha = \frac{R\tan\alpha}{2\pi} \qquad (3.6)$$

Using formulas (3.3), (3.4), (3.5) and (3.6) the membrane forces due to dead and dome load acting on the shell can be calculated. For the numerical values see table 3.1 and 3.2.

TABLE 3.1 MEMBRANE FORCES DUE TO DEAD LOAD

Y	N_ϕ	N_θ
65.45	-13819.70800000	5668.13570000
70.45	-11874.37800000	6101.14840000
75.45	-10120.60900000	6534.16100000
80.45	-8522.68420000	6967.17370000
85.45	-7053.24710000	7400.18640000
90.45	-5690.98930000	7833.19900000
95.45	-4419.06770000	8266.21170000
100.45	-3223.99140000	8699.22430000
105.45	-2094.83130000	9132.23700000
110.45	-1022.63450000	9565.24970000
115.45	0.00000000	9998.26230000

TABLE 3.2 MEMBRANE FORCES DUE TO DOME LOAD P

Y	N_ϕ	N_θ
65.45	-26588.85200000	753546.23000000
70.45	-24701.77700000	753546.23000000
75.45	-23064.81500000	753546.23000000
80.45	-21631.32700000	753546.23000000
85.45	-20365.59700000	753546.23000000
90.45	-19239.80400000	753546.23000000
95.45	-18231.95700000	753546.23000000
100.45	-17324.44300000	753546.23000000
105.45	-16502.99000000	753546.23000000
110.45	-15755.91000000	753546.23000000
115.45	-15073.54100000	753546.23000000

Displacement from membrane theory

From Eq. (1.16), for $r_1 = \infty$, $r_1 d\phi = dy$, $r_2 = y \operatorname{Tan}\alpha$, $N'_y = N'_\phi$, therefore obtains

$$\Delta\phi = \frac{\operatorname{Tan}\alpha}{r_1 Et}\left[r_1(N'_y - \nu N'_\theta) + r_2(\nu N'_y - N'_\theta)\right] - \frac{y \operatorname{Tan}\alpha}{Et}\frac{d}{dy}(N'_\theta - \nu N'_y)$$

i.e.

$$\Delta\phi = \frac{\operatorname{Tan}\alpha}{Et}\left[N'_y - \nu N'_\theta\right] - \frac{y \operatorname{Tan}\alpha}{Et}\frac{d}{dy}(N'_\theta - \nu N'_y) \tag{3.7}$$

For the uniform load acting on the shell, the Eq. (3.3) and (3.4) can be substituted into Eq. (3.7), then

$$\Delta\phi = \frac{q \operatorname{Tan}\alpha}{2Ety \operatorname{Cos}\alpha}\left[y(y - \nu) - L^2(1 + \nu)\right] \tag{3.8}$$

The horiaontal displacement ΔH, can be derived directly from Eq. (1.7)

$$\Delta H = \frac{y \operatorname{Sin}\alpha}{Et}(N'_\theta - \nu N'_y) \tag{3.9}$$

Substitute Eq. (3.3) and (3.4) into Eq. (3.9), thus

$$\Delta H = \frac{qy \operatorname{Sin}\alpha}{Et}\left[y \operatorname{Sin}\alpha \operatorname{Tan}\alpha + \frac{\nu(L^2 - y^2)}{2y \operatorname{Cos}\alpha}\right] \tag{3.10}$$

Using the same procedure, $\Delta\phi$ and ΔH due to dome load which acts at top edge of the shell can be derived as:

$$\Delta\phi = \frac{R \operatorname{Tan}\alpha}{2\pi Et}\left[\frac{1}{y \operatorname{Sin}\alpha \operatorname{Cos}\alpha}(\nu - 1) - \operatorname{Tan}\alpha\right] \tag{3.11}$$

$$\Delta H = \frac{Ry \operatorname{Sin}\alpha}{2\pi Et}\left(\operatorname{Tan}\alpha + \frac{\nu}{y \operatorname{Sin}\alpha \operatorname{Cos}\alpha}\right) \tag{3.12}$$

Table 3.3 and 3.4 show the numerical solution of the Eq. (3.8) to (3.12).

Bending of the bottom conical shell wall

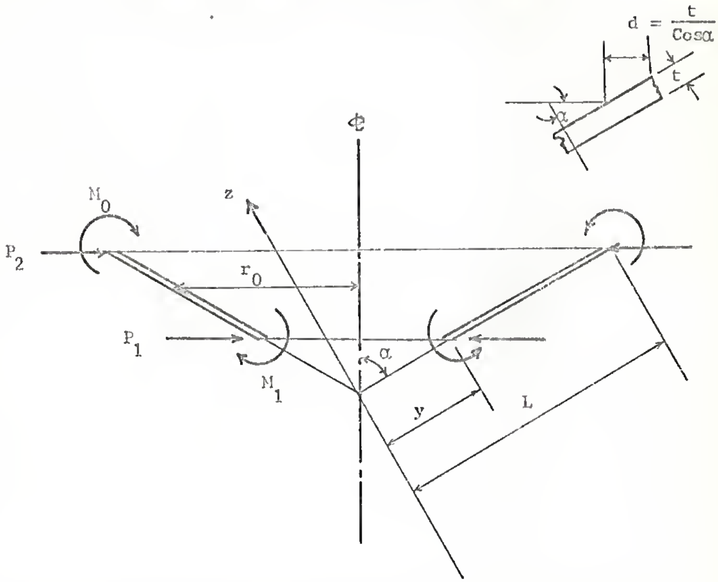


Fig. 3-3 Section through the conical shell wall (Showing the relation of the edge force)

The bottom wall of conical shell will be regarded as consisting of a large number of longitudinal beams supported on transverse elastic rings whose diameter increases in proportion to the distance from the apex of the cone. The bending analysis can be treated as the problem of bending of a beam on a elastic foundation. The shell as shown in Fig. 3-3 will be assumed to be under the action of an edge loading consisting of horizontal force P and moment M_y uniformly distributed along the edge circle of the shell. The modulus of the foundation furnished by the hoop rings, per unit length of circumference of the rings $K_0 = \frac{Ed}{r_0^2}$, where $r_0 = y \operatorname{Sin}\alpha$, and

TABLE 3.3 DISP. & ROTATION DUE TO MEMBRANE THEORY (D.L.)

Y	E*ΔH	E*Δφ
65.45	513530.59000000	-15512.81400000
70.45	540011.47000000	-13577.76900000
75.45	568440.92000000	-11841.80000000
80.45	598818.96000000	-10267.79100000
85.45	631145.55000000	-8827.31020000
90.45	665420.69000000	-7498.21220000
95.45	701644.37000000	-6262.99430000
100.45	739816.60000000	-5107.63700000
105.45	779937.40000000	-4020.78170000
110.45	822006.78000000	-2993.12430000
115.45	866024.70000000	-2016.97350000

TABLE 3.4 DISP. & ROTATION DUE TO MEMBRANE THEORY (DOME L.)

Y	E*ΔH	E*Δφ
65.45	37441834.00000000	-93775.96900000
70.45	40267633.00000000	-92468.56300000
75.45	43093430.00000000	-91334.43900000
80.45	45919229.00000000	-90341.29500000
85.45	48745028.00000000	-89464.36900000
90.45	51570827.00000000	-88684.39200000
95.45	54396626.00000000	-87986.14100000
100.45	57222425.00000000	-87357.39600000
105.45	60048224.00000000	-86788.27200000
110.45	62874023.00000000	-86270.68400000
115.45	65699822.00000000	-85797.92200000

$d = t/\text{Cos}\,\alpha$ is the thickness of the rings in the direction normal to the axis of the cone. Hence the modulus per unit length of the longitudinal beams will be $K = bK_0$ where $b = b_0 y$ is the width of the beams increasing linearly with the distance from the appex. Thus

$$K = b_0 y K_0 = \frac{b_0 Et}{y\,\text{Sin}^2\alpha\,\text{Cos}\,\alpha}$$

The flexural rigidity of the beam will be

$$D = EI = D_0 b_0 y = \frac{b_0 y t^3 E}{12(1-\nu^2)\text{Cos}^3\alpha}$$

Putting these values of K and EI into the differential equation of bending

$$\frac{d^2}{dy^2}\left(EI\frac{d^2Z}{dy^2}\right) + KZ = 0 \qquad (3.13)$$

i.e.

$$\frac{d^2}{dy^2}\,\frac{b_0 y t^3 E}{12(1-\nu^2)\text{Cos}^3\alpha}\,\frac{d^2Z}{dy^2} + \frac{b_0 Et}{y\,\text{Sin}^2\alpha\,\text{Cos}\,\alpha} = 0 \qquad (3.14)$$

After rearranging and differenting of equation (3.14), yields

$$y\frac{d^4Z}{dy^4} + 2\frac{d^3Z}{dy^3} + \frac{12Z}{t^2y}\text{Cot}^2\alpha = 0 \qquad (3.15)$$

The finite difference equations will therefore be

$$y\left(\frac{Z_{i+2} - 4Z_{i+1} + 6Z_i - 4Z_{i-1} + Z_{i-2}}{h^4}\right) + 2\left(\frac{Z_{i+2} - 2Z_{i+1} + 2Z_{i-1} - Z_{i-2}}{2h^3}\right)$$

$$+ \frac{12Z_i(1-\nu^2)}{t^2y_i}\text{Cot}^2\alpha = 0 \qquad (3.16)$$

Collecting similar terms of Z's

$$Z_{i-2}\left(\frac{y_i}{h^4}-\frac{1}{h^3}\right)+Z_{i-1}\left(\frac{2}{h^3}-\frac{4y_i}{h^4}\right)+Z_i\left(\frac{6y_i}{h^4}+\frac{12\cot^2\alpha\,(1-\nu^2)}{y_i t^2}\right)$$

$$+Z_{i+1}\left(\frac{-4y_i}{h^4}-\frac{2}{h^3}\right)+Z_{i+2}\left(\frac{y_i}{h^4}+\frac{1}{h^3}\right) = 0 \qquad (3.17)$$

Let

$$\frac{y_i}{h^4}-\frac{1}{h^3} = A_i$$

$$\frac{2}{h^3}-\frac{4y_i}{h^4} = B_i$$

$$\frac{6y_i}{h^4}+\frac{12\cot^2\alpha(1-\nu^2)}{y_i t^2} = C_i$$

$$\frac{-4y_i}{h^4}-\frac{2}{h^3} = D_i$$

$$\frac{y_i}{h^4}+\frac{1}{h^3} = E_i$$

Then Eq. (3.17) becomes

$$A_i Z_{i-2} + B_i Z_{i-1} + C_i Z_i + D_i Z_{i+1} + E_i Z_{i+2} = 0$$

In this problem $h = 5'$, total there are eleven points. Therefore there are eleven equations with fifteen unknowns. These eleven equations are:

$$A_3Z_1 + B_3Z_2 + C_3Z_3 + D_3Z_4 + E_3Z_5 = 0$$

$$A_4Z_2 + B_4Z_3 + C_4Z_4 + D_4Z_5 + E\ Z_6 = 0 \qquad (3.18)$$

$$\vdots$$

$$A_{13}Z_{11} + B_{13}Z_{12} + C_{13}Z_{13} + D_{13}Z_{14} + E_{13}Z_{15} = 0$$

Assuming zero displacement and rotation at bottom edge, (point 3)

$$Z_3 = 0$$

$$- Z_2 + Z_4 = 0 \qquad (3.19)$$

Assuming unit horizontal displacement and zero rotation at top edge (point 13), gets

$$Z_{13} = 2$$

$$- Z_{12} + Z_{14} = 0 \qquad (3.20)$$

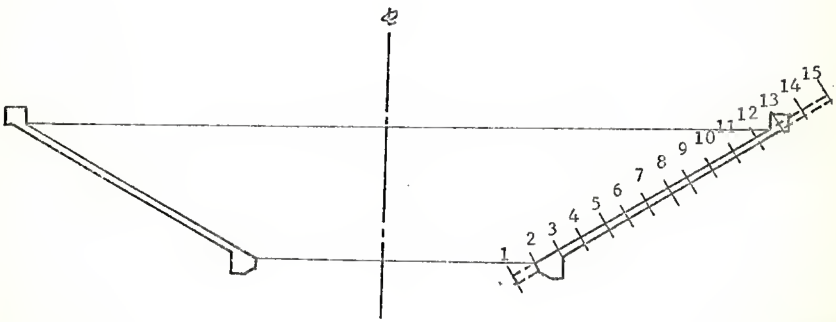


Fig. 3-4 The division of the conical shell wall in force analysis

By means of above assumption and combining Eq. (3.18), (3.19) and (3.20), there are a total of fifteen equations with fifteen unknowns. Hence these equations can be solved and get the displacement of each point corresponding to the unit horizontal displacement at top edge (point 13). Furthermore, to use these value, calculate the stiffness of the beam due to this displacement.

With the same concept, assuming bottom edge fixed and zero displacement and unit rotation at top edge, it is seen that

$$
\begin{aligned}
&Z_3 = 0 \\
&- Z_2 + Z_4 = 0 \\
&Z_{13} = 0 \\
&- Z_{12} + Z_{14} = 10
\end{aligned}
\tag{3.21}
$$

Eq. (3.21) together with Eq. (3.18) will provide another set of solutions. Then the stiffness of the beam due to rotation can be solved. Using the same procedure the stiffness at bottom edge can be determind.

In solving the above equations, use the same method as before. Write the equation in matrix form

$$
\begin{bmatrix}
A_3 & B_3 & C_3 & D_3 & E_3 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & A_4 & B_4 & C_4 & D_4 & E_4 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & A_5 & B_5 & C_5 & D_5 & \cdots & 0 & 0 & 0 & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot & \cdot \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & B_{13} & C_{13} & D_{13} & E_{13} \\
0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & -1 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ \cdot \\ \cdot \\ \cdot \\ Z_{11} \\ Z_{12} \\ Z_{13} \\ Z_{14} \\ Z_{15} \end{bmatrix}
=
\overset{G_1}{\begin{Bmatrix} 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \\ 0 \\ 2 \\ 0 \end{Bmatrix}}
\text{or}
\overset{G_2}{\begin{Bmatrix} 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \\ 0 \\ 0 \\ 10 \end{Bmatrix}}
\text{or}
\overset{G_3}{\begin{Bmatrix} 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}}
\text{or}
\overset{G_4}{\begin{Bmatrix} 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \\ 10 \\ 0 \\ 0 \end{Bmatrix}}
$$

Let the coefficient matrix be A, the matrix of constant term of the equation be G, then

$$\left\{ Z \right\} = \left[A \right]^{-1} \left\{ G \right\}$$

The A, A^{-1} and the solution of those equations see table (3.5), (3.6), (3.7), (3.8) and (3.9).

Forces due to edge effect

From above analysis there are already obtained the displacement due to edge effect. Now use this value of displacement to calculate the bending moments and forces both at meridional and hoop directions. From general theory, the moment is equal the second direvative of displacement in Z direction with respect to y, therefore the moment at meridinal direction can be obtained

TABLE 3.5 COEFFICIENT MATRIX OF AX=G

.096720	-.402880	.760329	-.434880
.112720	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.104720	-.434880	.798960	-.466880
.120720	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.112720	-.466880	.838833	-.498880
.128720	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.120720	-.498880	.879716	-.530880
.136720	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.128720	-.530880	.921432	-.562880
.144720	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.136720	-.562880	.963842	-.594880
.152720	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000

.144720	-.594880	1.006839	-.626880
.160720	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.152720	-.626880	1.050333	-.658880
.168720	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.160720	-.658880	1.094255	-.690880
.176720	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.168720	-.690880	1.138545	-.722880
.184720	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.176720	-.722880	1.183158	-.754880
.192720	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
-1.000000	0.000000	1.000000	0.000000
.000000	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
1.000000	0.000000	0.000000	0.000000
-1.000000	0.000000	1.000000	0.000000

.000000	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
1.000000	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.000000	0.000000	0.000000	0.000000
.000000			

TABLE 3.6 INVERSION OF MATRIX A

.10339123E+02	-0.27975251E+01	-0.30169850E+01
.13062459E+02	0.74206158E+01	0.26669520E+01
.30304978	-0.44017222	-0.45849873
- .27854978	-0.11628985	-0.25199331E-01
.00000000	0.46548200E-02	0.23346486E-02
.00000000	-0.82443642	0.58797084
.16765051E+01	0.12518429E+01	0.57518303
.15216664	-0.22554037E-01	-0.60050102E-01
- .45649217E-01	-0.22363885E-01	-0.58558356E-02
.00000000	0.10816901E-02	-0.28092077E-03
.00000000	0.00000000	0.10000000E+01
.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000
.00000000	0.17556360	0.58797084
.16765051E+01	0.12518429E+01	0.57518303

.15216664	-0.22554037E-01	-0.60050102E-01
- .45649217E-01	-0.22363885E-01	-0.58558356E-02
.00000000	0.10816901E-02	-0.28092077E-03
.00000000	0.13109301	0.21337940
.12518428E+01	0.29366761E+01	0.19864947E+01
.87090266	0.21006576	-0.52888221E-01
- .10026390	-0.66430234E-01	-0.21899456E-01
.00000000	0.40452676E-02	-0.40911268E-02
.00000000	0.60233167E-01	0.26217861E-01
.57518301	0.19864946E+01	0.33770157E+01
.22291010E+01	0.98125517	0.24497700
- .46048117E-01	-0.89505870E-01	-0.40859960E-01
.20000000E-08	0.75476550E-02	-0.13719374E-01
.00000000	0.15934886E-01	-0.31993933E-01
.15216663	0.87090267	0.22291011E+01
.35158160E+01	0.23523038E+01	0.10763078E+01
.30934961	0.48937200E-02	-0.36887910E-01
.00000000	0.68139390E-02	-0.27530351E-01
.00000000	-0.23618594E-02	-0.33486908E-01
- .22554035E-01	0.21006578	0.98125516
.23523037E+01	0.36065352E+01	0.24630225E+01
.11719418E+01	0.37227010	0.44937280E-01
.00000000	-0.83008150E-02	-0.33303301E-01
.00000000	-0.62884465E-02	-0.20153025E-01
- .60050099E-01	-0.52888184E-01	0.24497700
.10763079E+01	0.24630225E+01	0.36726315E+01

.25088310E+01	0.11475138E+01	0.27923684
.10000000E-09	-0.51580633E-01	-0.93392267E-03
.00000000	-0.47803859E-02	-0.85501821E-02
- .45649216E-01	-0.10026388	-0.46048117E-01
.30934961	0.11719418E+01	0.25088310E+01
.35575475E+01	0.22185381E+01	0.70470680
- .20000000E-07	-0.13017348	0.11735843
.00000000	-0.23419459E-02	-0.22375902E-02
- .22363884E-01	-0.66430230E-01	-0.89505880E-01
.48936700E-02	0.37227011	0.11475138E+01
.22185381E+01	0.28381276E+01	0.11989235E+01
.00000000	-0.22146511	0.36512378
.00000000	-0.61322310E-03	-0.78079200E-04
- .58558360E-02	-0.21899460E-01	-0.40859970E-01
- .36887930E-01	0.44937296E-01	0.27923690
.70470680	0.11989233E+01	0.12918149E+01
.00000000	-0.23862410	0.72195348
.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.00000000
.00000000	0.00000000	-0.10000000E-06
.00000000	0.00000000	0.00000000
.00000000	0.00000000	0.10000000E+01
.00000000	-0.61322310E-03	-0.78079200E-04
- .58558360E-02	-0.21899460E-01	-0.40859970E-01
- .36887930E-01	0.44937296E-01	0.27923690
.70470680	0.11989233E+01	0.12918149E+01

.0[illegible]	0.76137590	0.72195348
.00000[illegible]	-0.25546282E-02	0.14531162E-02
- .24394844E-01	-0.10700803	-0.23123572
- .28734082	0.32117190E-02	0.10889186E+01
.33692797E+01	0.65907381E+01	0.88061370E+01
.51888751E+01	0.22903086E+01	-0.93819192

TABLE 3.7 CONSTANT MATRIX G

G_1	0.0	10.0	0.0	0.0	0.0	0.0
	.0	0.0	0.0	0.0	0.0	0.0
	.0	0.0	0.0			
G_2	0.0	0.00	2.0	0.0	0.0	0.0
	.0	0.0	0.0	0.0	0.0	0.0
	.0	0.0	0.0			
G_3	0.0	0.0	0.0	0.0	0.0	0.0
	.0	0.0	0.0	0.0	0.0	0.0
	.0	10.0	0.0			
G_4	0.0	0.0	0.0	0.0	0.0	0.0
	.0	0.0	0.0	0.0	0.0	0.0
	.0	0.0	2.0			

TABLE 3.8 W VECTOR FOR TOP EDGE FIXED $\Delta\phi$ =1 OR ΔH=1 AT BOTTOM EDGE

Y	$\Delta\phi$ =1	ΔH=1
55.45	-27.97525100	-6.03397000
60.45	-8.24436420	1.17594170

65.45	0.00000000	2.00000000
70.45	1.75563600	1.17594170
75.45	1.31093010	.42675880
80.45	.60233167	.05243572
85.45	.15934886	-.06398787
90.45	-.02361859	-.06697382
95.45	-.06288447	-.04030605
100.45	-.04780386	-.01710036
105.45	-.02341946	-.00447518
110.45	-.00613223	-.00015616
115.45	0.00000000	0.00000000
120.45	-.00613223	-.00015616
125.45	-.02554628	.00290623

TABLE 3.9 W VECTOR FOR BOTTOM EDGE FIXED $\Delta\Phi$=1 OR ΔH=1 AT TOP EDGE

Y	$\Delta\Phi$=1	ΔH=1
55.45	.04654820	.00466930
60.45	.01081690	-.00056184
65.45	0.00000000	0.00000000
70.45	.01081690	-.00056184
75.45	.04045268	-.00818225
80.45	.07547655	-.02743875
85.45	.06813939	-.05506070
90.45	-.08300815	-.06660660
95.45	-.51580633	-.00186785

100.45	-1.30173480	.23471686
105.45	-2.21465110	.73024756
110.45	-2.38624100	1.44390700
115.45	0.00000000	2.00000000
120.45	7.61375900	1.44390700
125.45	22.90308600	-1.87638380

$$M_y = - D_0 \frac{d^2 Z}{dy^2} \quad (3.21)$$

Where D_0 is flexural rigidity of the shell

$$D_0 = \frac{Et^3}{12(1 - \nu^2) \, Cos^3 \alpha} \quad (3.21)$$

The hoop bending moment will be

$$M_\theta = \nu M_y \quad (3.22)$$

The shearing force Q_y can also be expressed in terms of the meridional bending moment M_y by a consideration of equilibrium of the meridional beam (see Fig. 3-3).

$$Q_y = - \frac{dy \, M_y}{dy} = D_0 (y \frac{d^3 Z}{dy^3} + \frac{d^2 Z}{dy^2}) \quad (3.24)$$

The meridional force N_y can be obtained as a component of the shearing force Q_y:

$$N_y = Q_y \, Tan \alpha = D_0 \, Tan \alpha \, (\frac{d^3 Z}{dy^3} + \frac{d^2 Z}{dy^2}) \quad (3.25)$$

The hoop force N will be proportional to the deflection Z and according to Hook's law, its values per unit length of the generator will be

$$N_\theta = \frac{Et}{y_i \, Tan \alpha} Z_i \quad (3.26)$$

Again write the equation (3.21), (3.23), (.24) and (3.25) in finite difference

$$M_y = -D_0\left(\frac{Z_{i-1} - 2Z_i + Z_{i+1}}{h^2}\right)$$

$$M_\theta = -\nu D_0\left(\frac{Z_{i-1} - 2Z_i + Z_{i+1}}{h^2}\right)$$

$$Q_y = D_0\left(\frac{Z_{i+2} - 2Z_{i+1} + 2Z_{i-1} - Z_{i-2}}{2h^3} + \frac{Z_{i-1} + 2Z_i + Z_{i+1}}{h^2}\right) \qquad (3.27)$$

$$N_y = D_0 \operatorname{Tan}\alpha\left(\frac{Z_{i+2} - 2Z_{i+1} + 2Z_{i-1} - Z_{i-2}}{2h^3} + \frac{Z_{i-1} - 2Z_i + Z_{i+1}}{h^2}\right)$$

Using the data of Z value obtained from former calculation, the M_y, M_θ, Q_y, N_y, N_θ can be obtained as shown in table (3.10) to (3.13).

TABLE 3.10 FORCES DUE TO EDGE EFFECT FOR TOP EDGE FIXED

AND $\Delta H=0, \Delta\phi=1$ AT BOTTOM EDGE

Y	Q	N_ϕ	N_θ	M_ϕ	M_θ
65.45	1.95377680	3.384[illegible]397[illegible]	0.00000000	.00010094	.00002019
70.45	.83530296	1.44678670	.00959184	.02899987	.00579997
75.45	.16174812	.28015588	.00668758	.02167972	.00433594
80.45	-.07441295	-.12888698	.00288176	.01001572	.00200314
85.45	-.09731782	-.16855936	.00071777	.00272393	.00054479
90.45	-.05916194	-.10247146	-.00010051	-.00028784	-.00005757
95.45	-.02314657	-.04009102	-.00025358	-.00093418	-.00018684
100.45	-.00344401	-.00596519	-.00018317	-.00068594	-.00013719
105.45	.00343091	.00594251	-.00008548	-.00028456	-.00005691
110.45	.00373356	.00646671	-.00002137	0.00000000	0.00000000
115.45	.00192227	.00332947	0.00000000	.00010094	.00002019

TABLE 3.11 FORCES DUE TO EDGE EFFECT FOR TOP EDGE FIXED

AND $\Delta H=1, \Delta\phi=0$ AT BOTTOM EDGE

Y	Q	N_ϕ	N_θ	M_ϕ	M_θ
65.45	.67595451	1.17078720	.01176167	.03292389	.00658478
70.45	.05940799	.10289763	.00642471	.01935935	.00387187
75.45	-.10623272	-.18400041	.00217707	.00702730	.00140546
80.45	-.09261803	-.16041909	.00025087	.00086570	.00017314
85.45	-.04565829	-.07908246	-.00028823	-.00105071	-.00021014
90.45	-.012913[illegible]5	-.02236606	-.00028500	-.00109986	-.00021997
95.45	.00150582	.00260816	-.00016253	-.00066089	-.00013218
100.45	.00490867	.00850207	-.00006552	-.00027891	-.00005578

105.45	.0039499[illegible]	.00684143	-.00001633	-.00007109	-.00001422
110.45	.00224562	.00388952	-.00000054	0.00000000	0.00000000
115.45	.00112403	.00194688	0.00000000	.00000257	.00000051

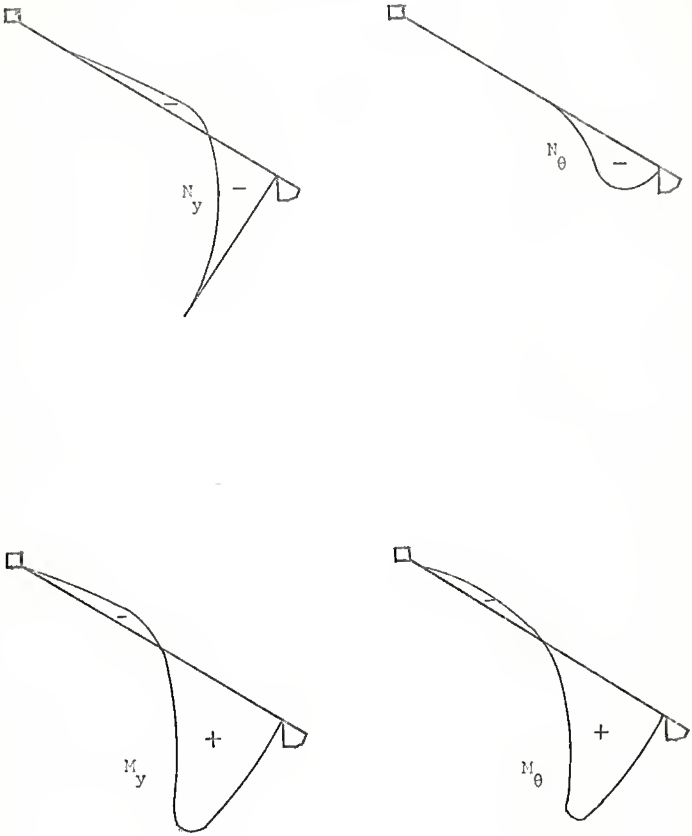
TABLE 3.12 FORCES DUE TO EDGE EFFECT FOR BOTTOM EDGE FIXED AND $\Delta H=0$, $\Delta\phi=1$ AT TOP EDGE

Y	G	N_ϕ	N_θ	M_ϕ	M_θ
65.45	-.00265773	-.00460333	0.00000000	-.04302420	-.00860484
70.45	-.00132525	-.00229540	.00005910	-.04284615	-.00856923
75.45	-.00342186	-.00592683	.00020637	-.04235833	-.00847167
80.45	-.01032255	-.01787917	.00036111	-.04178181	-.00835636
85.45	-.02029657	-.03515467	.00030693	-.04190258	-.00838052
90.45	-.02420246	-.04191988	-.00035323	-.04439057	-.00887811
95.45	-.00035448	-.00061398	-.00207998	-.05151472	-.01030294
100.45	.08558652	.14[illegible]24015	-.00498794	-.06445162	-.01289032
105.45	.26565265	.46012367	-.00808364	-.07947863	-.01589577
110.45	.52755795	.91375691	-.00831567	-.08230331	-.01646066
115.45	.73974176	1.28126990	0.00000000	-.04302420	-.00860484

TABLE 3.13 FORCES DUE TO EDGE EFFECT FOR BOTTOM EDGE FIXED AND $\Delta H=1$, $\Delta\phi=0$ AT TOP EDGE

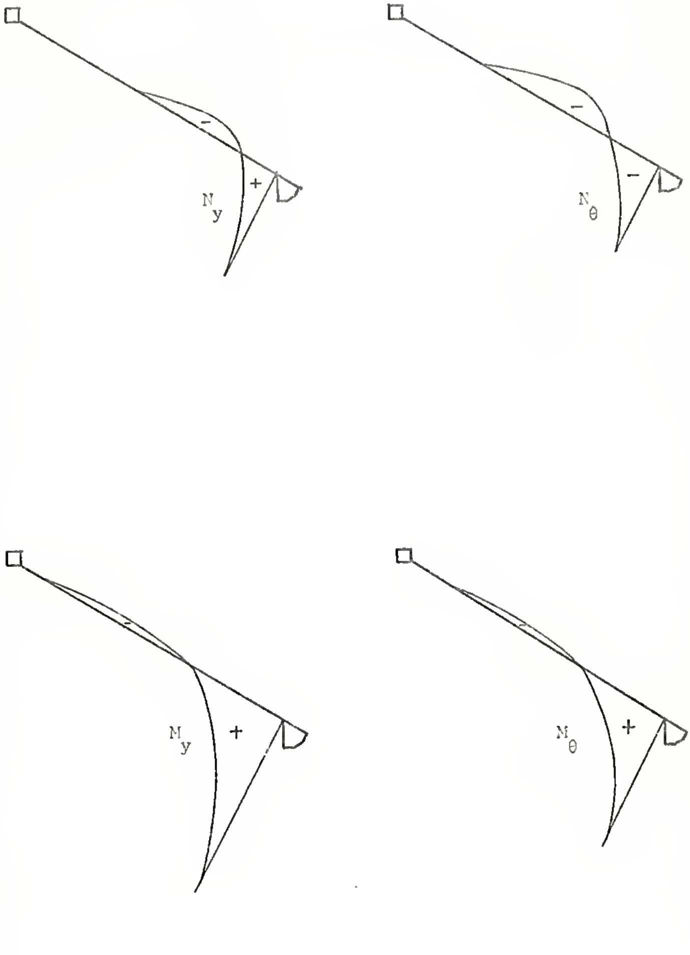
Y	G	N_ϕ	N_θ	M_ϕ	M_θ
65.45	-.000953[illegible]5	-.00165[illegible]74	0.00000000	-.02376767	-.00475353
70.45	-.00063968	-.00110796	-.00000307	-.02377691	-.00475538
75.45	-.00024427	-.00042308	-.00004174	-.02390235	-.00478047

80.45	.00157743	.00273218	-.00013126	-.02421933	-.00484387
85.45	.00620787	.01075235	-.00024802	-.02467400	-.00493480
90.45	.01371832	.02376082	-.00028344	-.02486405	-.00497281
95.45	.02007407	.03476929	-.00000753	-.02379841	-.00475968
100.45	.01339599	.02320253	.00089938	-.01990407	-.00398081
105.45	-.028181[illegible]9	-.04881106	.00266546	-.01174731	-.00234946
110.45	-.13168065	-.22807751	.00503178	0.00000000	0.00000000
115.45	-.30976045	-.53652068	.00666783	.00915366	.00183073

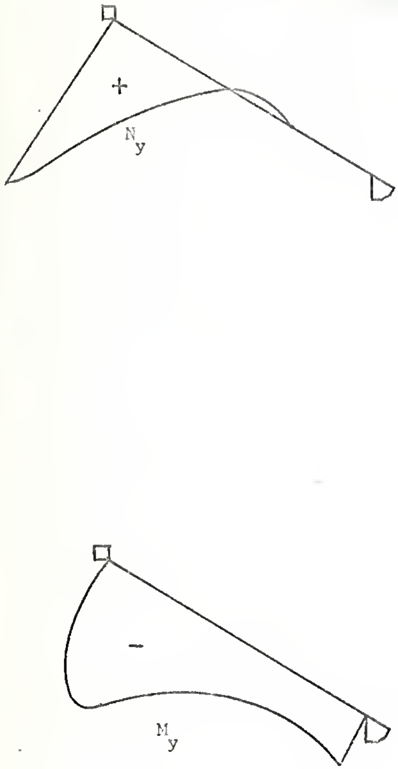


Dia. 3-3 Top edge fixed, and $\Delta H = 0$, $\Delta\phi = 1$ at bottom edge

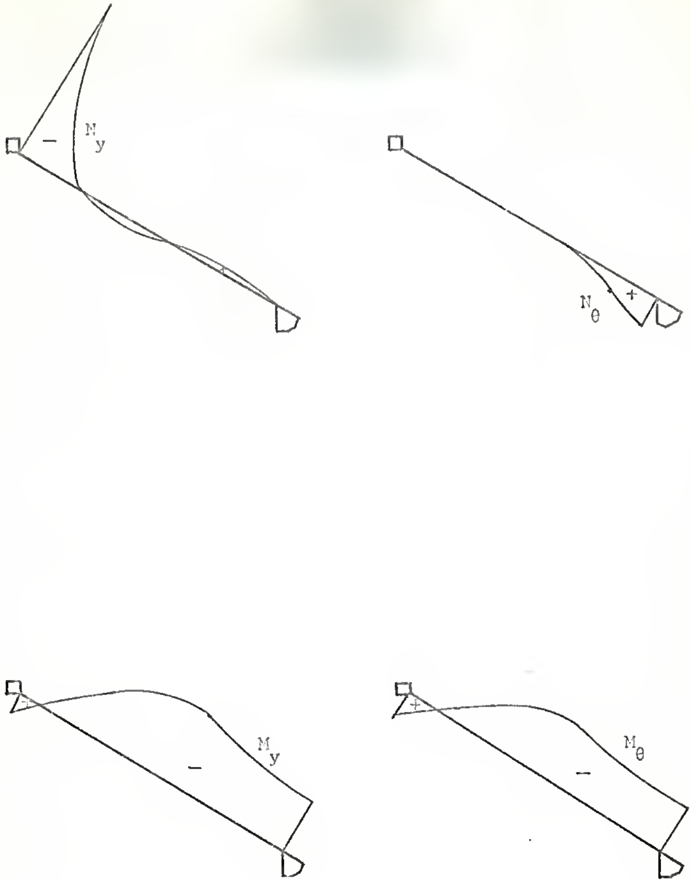
* The Dia. on above and the following pages are just for the purpose of showing the pattern of forces. Therefore no scale and dimension are presented.



Dia. 3-5 Top edge fixed and $\Delta H = 1$, $\Delta\phi = 0$ at bottom edge



Dia. 3-6 Bottom edge fixed, and $\Delta H = 0$, $\Delta\phi = 1$ at top edge



Dia. 3-7 Bottom edge fixed, and $\Delta H = 1$, $\Delta \phi = 0$ at top edge

THE CALCULATION OF THE FLEXIBILITY OF THE DOME AND THE CONICAL SHELL WALL

In the previous section the stiffness of the dome and the conical shell wall had already obtained. The next step is to apply the reciprocal law to calculate the flexibility.

At first the dome is considered. Based on statically equilibrium condition, for unit horizontal displacement and zero rotation at edge of dome, we have

$$\begin{aligned} F_{11} H_{11} + F_{12} H_{21} &= 1 \\ F_{21} H_{11} + F_{22} H_{21} &= 0 \end{aligned} \qquad (4.1)$$

Where H represent stiffness, F is flexibility for corresponding relation, see Fig. 4-1.

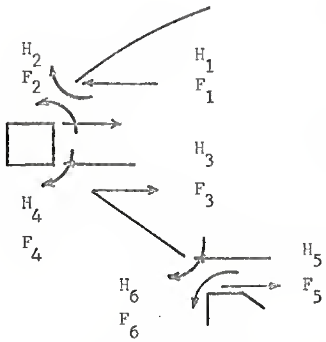


Fig. 4-1 The relation of stiffness & flexibility

Then assume there is a unit rotation and a zero displacement at the edge, hence

$$F_{11}H_{12} + F_{12}H_{22} = 0$$
$$F_{21}H_{12} + F_{22}H_{22} = 1 \qquad (4.2)$$

Combining Eq. (4.1) and (4.2), yields

$$\begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix} \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad (4.3)$$

Hence

$$\begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}^{-1} \qquad (4.4)$$

Using the same idea and procedure the flexibility matrix of the bottom conical wall can be obtained. For $\Delta H = 1, \Delta\phi = 0$ at top and let the bottom of the wall fixed, then

$$F_{33}H_{33} + F_{34}H_{43} + F_{35}H_{53} + F_{36}H_{63} = 1$$
$$F_{43}H_{33} + F_{44}H_{43} + F_{45}H_{53} + F_{46}H_{63} = 0$$
$$F_{53}H_{33} + F_{54}H_{43} + F_{55}H_{53} + F_{56}H_{63} = 0 \qquad (4.5)$$
$$F_{63}H_{33} + F_{64}H_{43} + F_{65}H_{53} + F_{66}H_{63} = 0$$

For $\Delta H = 0, \Delta\phi = 1$ at top and still let bottom edge fixed, obtains

$$F_{33}H_{34} + F_{34}H_{44} + F_{35}H_{54} + F_{36}H_{64} = 0$$
$$F_{43}H_{34} + F_{44}H_{44} + F_{45}H_{54} + F_{36}H_{64} = 1$$

(4.6)

$$F_{53}H_{34} + F_{54}H_{44} + F_{55}H_{54} + F_{56}H_{64} = 0$$

$$F_{63}H_{34} + F_{64}H_{44} + F_{65}H_{54} + F_{66}H_{64} = 0$$

For $\Delta H = 0, \Delta\phi = 0$ at top and $\Delta H = 1, \Delta\phi = 0$ at bottom of the conical shell wall, hence

$$\begin{aligned} F_{33}H_{35} + F_{34}H_{45} + F_{35}H_{55} + F_{36}H_{65} &= 0 \\ F_{43}H_{35} + F_{44}H_{45} + F_{45}H_{55} + F_{36}H_{65} &= 0 \\ F_{53}H_{35} + F_{54}H_{45} + F_{55}H_{55} + F_{56}H_{65} &= 1 \\ F_{63}H_{35} + F_{64}H_{45} + F_{65}H_{55} + F_{66}H_{65} &= 0 \end{aligned} \qquad (4.7)$$

For $\Delta H = 0, \Delta\phi = 0$ at top edge and $\Delta H = 0, \Delta\phi = 1$ at bottom edge of the conical shell wall, therefore

$$\begin{aligned} F_{33}H_{36} + F_{34}H_{46} + F_{35}H_{56} + F_{36}H_{66} &= 0 \\ F_{43}H_{36} + F_{44}H_{46} + F_{45}H_{56} + F_{46}H_{66} &= 0 \\ F_{53}H_{36} + F_{54}H_{46} + F_{55}H_{56} + F_{56}H_{66} &= 0 \\ F_{63}H_{36} + F_{64}H_{46} + F_{65}H_{56} + F_{66}H_{66} &= 1 \end{aligned} \qquad (4.8)$$

From Eq. (4.5), (4.6), (4.7) and (4.8) there are sixteen simultaneous equations with sixteen unknowns. Therefore it can be solved easily. Write it in matrix form:

$$\begin{bmatrix} F_{33} & F_{34} & F_{35} & F_{36} \\ F_{43} & F_{44} & F_{45} & F_{46} \\ F_{53} & F_{54} & F_{55} & F_{56} \\ F_{63} & F_{64} & F_{65} & F_{65} \end{bmatrix} \begin{Bmatrix} H_{33} & H_{34} & H_{35} & H_{36} \\ H_{43} & H_{44} & H_{45} & H_{46} \\ H_{53} & H_{54} & H_{55} & H_{56} \\ H_{63} & H_{64} & H_{65} & H_{66} \end{Bmatrix} = \begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{Bmatrix}$$

Hence

$$\begin{bmatrix} F_{33} & F_{34} & F_{35} & F_{36} \\ F_{43} & F_{44} & F_{45} & F_{46} \\ F_{53} & F_{54} & F_{55} & F_{56} \\ F_{63} & F_{64} & F_{65} & F_{66} \end{bmatrix} = \begin{Bmatrix} H_{33} & H_{34} & H_{35} & H_{36} \\ H_{43} & H_{44} & H_{45} & H_{46} \\ H_{53} & H_{54} & H_{55} & H_{56} \\ H_{63} & H_{64} & H_{65} & H_{66} \end{Bmatrix}^{-1} \tag{4.9}$$

The value of H's are already calculated in previous section. For the solution of Eq. (4.4) and (4.9) see table (4.1).

TABLE 4.1 VALUES OF FLEXIBILITY

F_{11}	0.24260895E+05	F_{12}	0.48091368E+03	F_{21}	-0.6261863E+04
F_{22}	-0.13341623E+03				
F_{33}	-0.92809630	F_{34}	-0.41100000E-02	F_{35}	0.17373000E-02
F_{36}	-0.23932046E+02				
F_{34}	0.51200690	F_{44}	-0.20574000E-01	F_{45}	-0.15800000E-03
F_{64}	-0.10017272E+02				
F_{53}	0.92809668	F_{54}	0.30379565E+02	F_{55}	-0.17375700E-02
F_{56}	-0.64433998E+01				
F_{63}	-0.32085288	F_{64}	-0.10510547E+02	F_{65}	0.34182137
F_{66}	0.22039434E+01				

COMBINATION ANALYSIS OF THE DOME-RING-CONICAL SHELL WALL

Due to membrane analysis both the dome and the shell wall induced a great deal of displacement and rotation along the edge. In order to restraint these displacements and rotations, a ring will be provided. This ring acts as a circular tension tie. And for the purpose of reducing the moment at edge of the shell, it may logically be prestressed.

Let the ring be in the rectangular shape. There are three forces acting on it, M_a, H, and M_x as shown in Fig. 5-1. Corresponding to those forces, the horizontal displacement H and the rotation ϕ are therefore induced. The relation of it are easily derived. The result are

$$H = \frac{r^2}{Ebd^2} H$$

$$H = \frac{12r^2 y}{Ebd^2} M_a \tag{5.1}$$

$$\alpha = \frac{12r^2}{Ebd^3} M_a$$

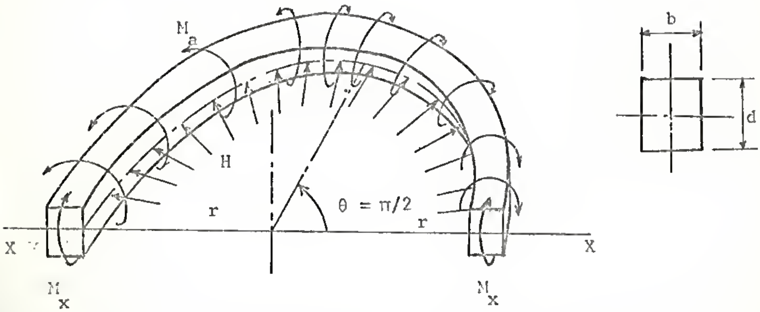


Fig. 5-1 The relation of the forces which acting on the ring

The relation among dome ring and shell wall are shown in Fig. 5-2.

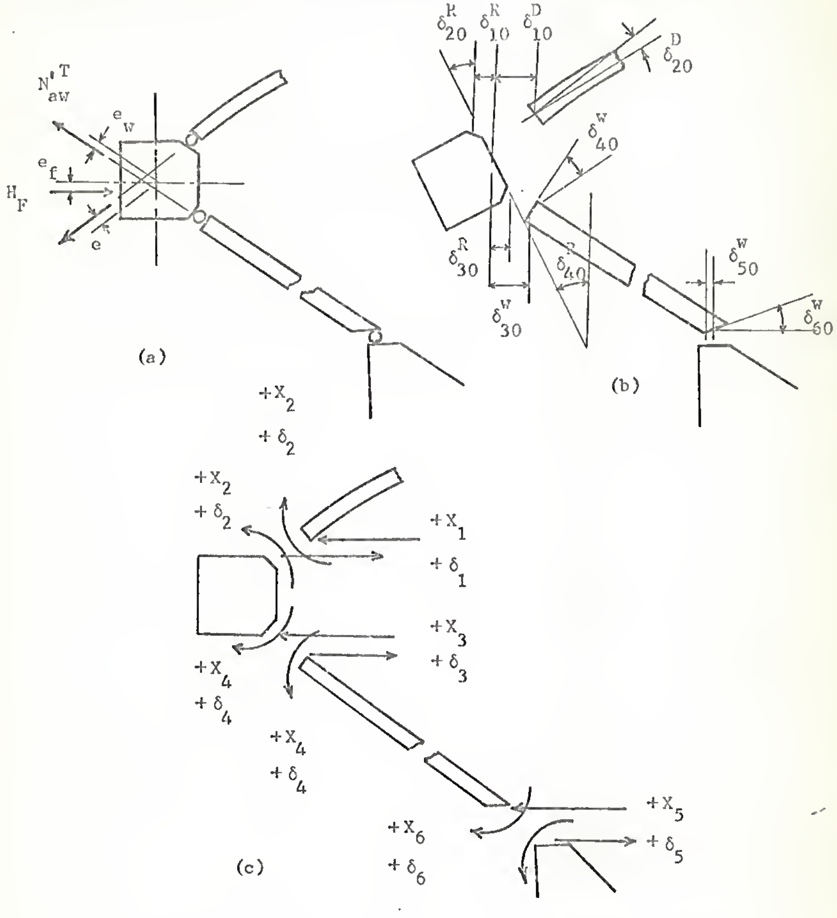


Fig. 5-2 The relation among the dome-ring-shell wall and the sign convention

Fig. 5-2a and b illustrate the system in which all stress resultants for the dome and shell wall are determined by the membrane theory where N'_{ad}, N'_{aw} and N'^{b}_{aw} are meridian forces at edges of dome and wall. H_f is the ring

force due to prestressing. In this problem the ring and dome supported on the conical shell wall will settle uniformly every where due to uniformly distributed load. Therefore it can be stated that the vertical settlement has no effect on the analysis of the relation between dome and ring. The effect of the vertical deflection of the bottom conical shell wall is already induced into the horizontal displacement δ_3 and δ_5 (Fig. 5-2c).

Now there will be ten components due to translation and rotation of: the dome, the top and the bottom of the ring, the top of the shell wall and the bottom of shell wall (δ^D_{10}, δ^D_{20}, δ^R_{10}, δ^R_{20}, δ^R_{30}, δ^R_{40}, δ^W_{30}, δ^W_{40}, δ^W_{50} and δ^W_{60} respectively), Fig. 5-2c illustrates this system and shows that, in addition to all the previous displacements, there may be an additional ring rotation for the center line of the wall does not intersect the centroid of the ring cross section

$$\delta^R_{20} = -\frac{6r^2e}{Ebd^2}N'_{aD} \qquad\qquad \delta^R_{40} = \frac{12r^2e^W}{Ebd^3}N'^T_{aw}$$

and

$$\delta^R_{10} = (\cos\alpha + \frac{12y_0e}{d^2})\frac{r^2N'_{aD}}{Ebd} = (\cos\alpha + \frac{6e}{d})\frac{r^2N'_{aD}}{Ebd}$$

Where α is the angle between the radius of curvature and the axis of revolution at the edge of the dome.

$$\delta^R_{30} = (\sin\alpha + \frac{12y_0e_W}{d^2})\frac{r^2N'^T_{aw}}{Ebd}$$

For δ^D_{10}, δ^D_{20}, δ^W_{30}, δ^W_{40}, δ^W_{50} and δ^W_{60} are already calculated in the section of membrane analysis. Then

$$\delta_{10} = \delta^D_{10} + \delta^R_{10}$$

$$\delta_{20} = \delta_{20}^{D} + \delta_{20}^{R}$$

$$\delta_{30} = \delta_{30}^{D} + \delta_{30}^{R}$$

$$\delta_{40} = \delta_{40}^{D} + \delta_{40}^{R}$$

In order to restrain these displacement due to membrane theory, there are four corrections: a forces X_1 and a moment X_2, which correspond to the required dome - ring values; and a force X_3, and a moment X_4, which correspond to the ring-wall values; and a force X_5, and a moment X_6, which correspond to the wall and the basement. The correction displacements due to those forces as shown in Fig. 5-2d. Consider first the horizontal displacement at the junction of the dome and ring, due to the force X_1, obtains

$$\delta_{11} = \delta_{11}^{D} + \delta_{11}^{R} = F_{11}X_1 + \frac{r^2X_1}{Ebd} - \frac{6r^2y_0}{Ebd^2}X_1$$

$$= (F_{11} - \frac{2r^2}{Ebd})X_1$$

$$\delta_{12} = \delta_{21}^{D} + \delta_{21}^{R} = (F_{21} - \frac{6r^2}{Ebd^2})X_1$$

From X_2 force, yields

$$\delta_{12} = \delta_{12}^{D} + \delta_{12}^{R} = (F_{12} - \frac{6r^2}{Ebd^2})X_2$$

$$\delta_{22} = \delta_{22}^{D} + \delta_{22}^{R} = (F_{22} + \frac{12r^2}{Ebd^3})X_2$$

And from the ring-wall forces X_3 and X_4 oome the displacements:

$$\delta_{13} = \delta_{13}^{R} = -\frac{r^2 X_3}{Ebd} - \frac{12r^2 y_0 (d/2) X_3}{Ebd}$$

$$= -\frac{r^2 X_3}{Ebd} + \frac{3r^2 X_3}{Ebd} = \frac{2r^2 X_3}{Ebd}$$

and

$$\delta_{14} = \delta_{14}^{R} = \frac{12r^2 y_0}{Ebd^3} X_4 = \frac{6r^2 X_4}{Ebd^2}$$

The rotation of the ring at this junction due to X_3 is

$$\delta_{23} = -\frac{12r^2 y_0}{Ebd^3} X_3 = -\frac{12r^2 d/2}{Ebd^3} X_3 = -\frac{6r^2}{Ebd^2} X_3$$

and the rotation due to X_4 will be

$$\delta_{24} = -\frac{12r^2}{Ebd^3} X_4$$

In a similar manner the displacements at the ring-wall junction can be derived.

$$\delta_{31} = \frac{2r^2 X_1}{Ebd}$$

$$\delta_{32} = -\frac{12r^2 d/2}{Ebd^3} X_2 = -\frac{12r^2}{Ebd^2} X_2$$

$$\delta_{33} = \delta_{33}^{R} + \delta_{33}^{W} = \frac{r^2 X_3}{Ebd} - \frac{6r^2 y_0 X_3}{Ebd^2} + F_{33} X_3$$

$$= F_{33} X_3 - \frac{2r^2}{Ebd} X_3$$

$$\delta_{34} = \delta_{34}^{R} + \delta_{34}^{w} = \frac{6r^2}{Ebd^2}X_4 + F_{34}X_4$$

and

$$\delta_{41} = \frac{12y_0r^2X_1}{Ebd^3} = \frac{6r^2}{Ebd^2}X_1$$

$$\delta_{42} = -\frac{12r^2}{Ebd^3}X_2$$

$$\delta_{43} = \delta_{43}^{R} + \delta_{43}^{w} = \frac{6r^2}{Ebd^2}X_3 + F_{43}X_3$$

$$\delta_{44} = \delta_{44}^{R} + \delta_{44}^{w} = \frac{12r^2}{Ebd^3}X_4 + F_{44}X_4$$

$$\delta_{45} = F_{45}X_5 \quad ; \quad \delta_{54} = F_{54}X_4$$

$$\delta_{53} = F_{53}X_3 \quad ; \quad \delta_{46} = F_{46}X_6$$

$$\delta_{64} = F_{64}X_4 \quad ; \quad \delta_{36} = F_{36}X_6$$

$$\delta_{63} = F_{63}X_3 \quad ; \quad \delta_{35} = F_{35}X_5$$

Now there will be six simultaneous compatibility equations to solve for the six corrections X_1, X_2, X_3, X_4, X_5 and X_6. Those six equations are:

$$\delta_{10} + \delta_{11} + \delta_{12} + \delta_{13} + \delta_{14} = 0$$

$$\delta_{20} + \delta_{21} + \delta_{22} + \delta_{23} + \delta_{24} = 0$$

$$\delta_{30} + \delta_{31} + \delta_{32} + \delta_{33} + \delta_{34} + \delta_{35} + \delta_{36} = 0$$

$$\delta_{40} + \delta_{41} + \delta_{42} + \delta_{43} + \delta_{44} + \delta_{45} + \delta_{46} = 0 \qquad (5.2)$$

$$\delta_{50} + \delta_{54} + \delta_{55} + \delta_{56} = 0$$

$$\delta_{60} + \delta_{63} + \delta_{64} + \delta_{65} + \delta_{56} = 0$$

i.e.

$$\left(\delta_{10}^{D} + \left(\operatorname{Cos}\alpha + \frac{6e}{d}\right)\frac{r^2 N'_{aD}}{Ebd}\right) + \left(F_{11} - \frac{2r^2}{Ebd}\right)X_1 + \left(F_{12} - \frac{6r^2}{Ebd^2}\right)X_2$$

$$+ \frac{2r^2}{Ebd}X_3 + \frac{6r^2}{Ebd^2}X_4 = 0$$

$$\left(\delta_{20}^{D} - \frac{6r^2}{Ebd^2}N'_{aD}\right) + \left(F_{21} - \frac{2r^2}{Ebd}\right)X_1 + \left(F_{22} + \frac{12r^2}{Ebd^3}\right)X_2$$

$$- \frac{6r^2}{Ebd^2}X_3 - \frac{12r^2}{Ebd^3}X_4 = 0 \qquad (5.3)$$

$$\left(\delta_{30}^{W} + \left(\operatorname{Sin}\alpha + \frac{6e_W}{d}\right)\frac{r^2 N'^{T}_{aW}}{Ebd}\right) + \frac{2r^2}{Ebd}X_1 - \frac{6r^2}{Ebd^2}X_2 + \left(F_{33} - \frac{2r^2}{Ebd}\right)X_3$$

$$+ \left(F_{34} + \frac{6r^2}{Ebd^2}\right)X_4 + F_{35}X_5 + F_{36}X_6 = 0$$

$$\left(\delta_{40}^{W} + \frac{12r^2 e_W}{Ebd^3}N'_{aw}\right) + \frac{6r^2}{Ebd^2}X_1 - \frac{12r^2}{Ebd^3}X_2 + \left(\frac{6r^2}{Ebd^2} + F_{43}\right)X_3$$

$$+ \left(F_{44} + \frac{12r^2}{Ebd^3}\right)X_4 + F_{45}X_5 + F_{46}X_6 = 0$$

$$\delta_{50}^{W} + F_{53}X_3 + F_{54}X_4 + F_{55}X_5 + F_{56}X_6 = 0$$

$$\delta_{60}^{W} + F_{63}X_3 + F_{64}X_4 + F_{65}X_5 + F_{66}X_6 = 0$$

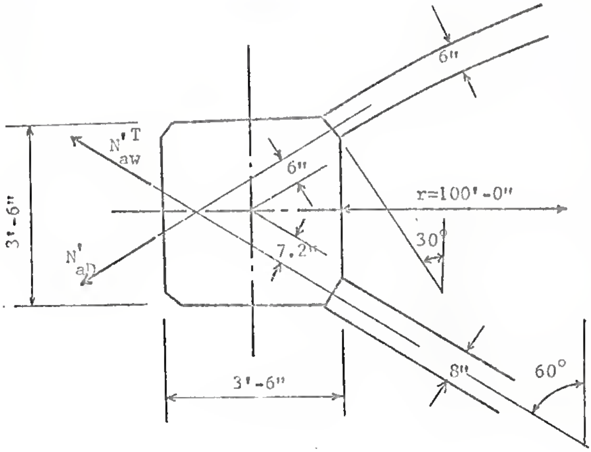


Fig. 5-3 Dimensions and the relation of the dome-ring and the conical shell wall

According the dimension shown in the Fig. 5-3, and the data already obtained from previous calculations, those equations can be solved. The coefficient matrix A, A^{-1} and the constant term matrix see table 5.1, 5.2 and 5.3. The X value is listed in table 5.4.

TABLE 5.1 COEF. MATRIX A

22630.19000000	1149.08600000	1630.00000000
1421.00000000	0.00000000	0.00000000
46318.63400000	933.41623000	-1421.00000000
800.00000000	0.00000000	0.00000000
1630.00000000	-1421.00000000	-1629.07200000
1421.00000000	.00173730	23.93204600
1421.00000000	-800.00000000	1420.48800000
799.98000000	-.00015800	-10.01727200
.00000000	0.00000000	.92809668
30.37956500	-.00173757	6.44339980
.00000000	0.00000000	.32085288
-10.51054700	.34182137	2.20394340

TABLE 5.2 INVERSION OF MATRIX A

-.00002348	.00003257	-.00001887
.00003779	.00012860	.00000077
.00058474	-.00026229	.00007396
-.00084751	-.00158927	-.00000885
-.00002478	.00000786	-.00026192
.00043882	.00165166	.00000993
.00063327	-.00031556	.00054121
-.00041992	-.00265737	-.00001645
.03865500	-.01926180	.03303850
-.02563610	-1.16112000	2.91942280
-.00297175	.00148150	-.00250509
.00190974	.16717563	.00086341

TABLE 5.3 CONST MATRIX

1504189.30000000
-16102173.00000000
-56657982.00000000
7325131.50000000
-38468895.00000000
124701.59000000

TABLE 5.4 X FORCES

X_1	-4160.34510000
X_2	55840.69700000
X_3	-45645.55300000
X_4	74517.98900000
X_5	43339679.00000000
X_6	-6303357.00000000

CONCLUSION AND DISCUSSIONS

After the edge forces X are obtained, the corresponding forces produced in the shell due to those X's can be achieved. Let φ_i represents the corresponding force, N_ϕ, N_θ, M_ϕ, M_θ, etc., of the shell, H_i represents the force due to unit corresponding displacement which was shown in table (3.7). Thus

$$\varphi_i = F_i X_i H_i \tag{5.4}$$

Combined the φ_i and the relative force from the membrane theory, the resultant force is obtained. In Fig. 6-1 are shown the forces at edge of the shell both due to membrane theory and due to the edge force X. Adding those two forces together, the total resultant forces can be obtained.

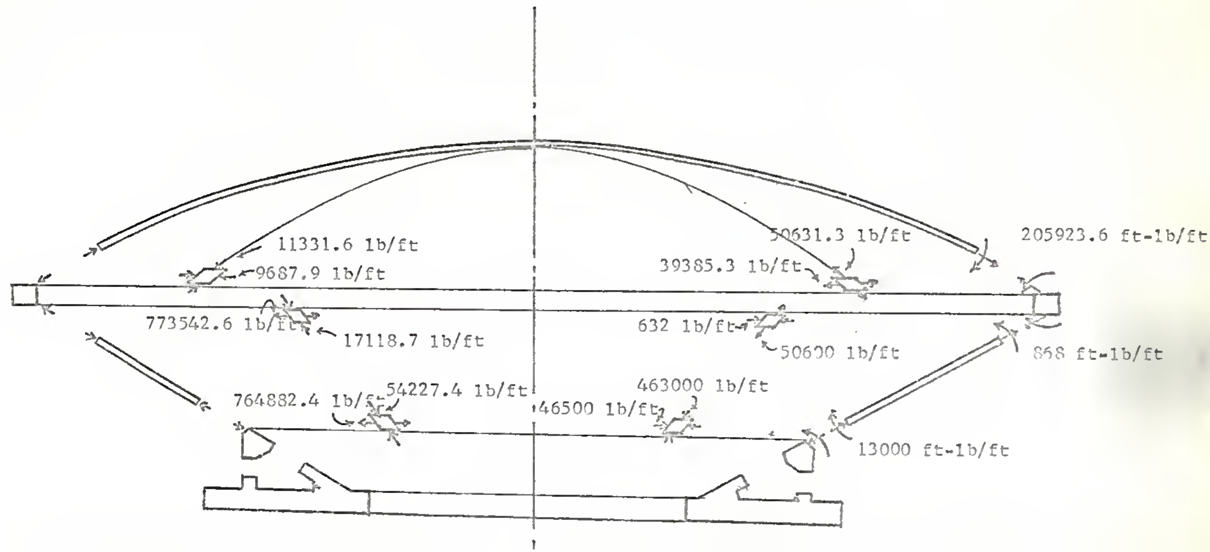
From the membrane theory the ring is in tension and the dome is entirely in compression; and the hoop force in the shell wall is in tension while the meridian force is in compression. The membrane ring tension is

$$T = H_a = N'_a r \cos \alpha$$

i.e.

$$T = (11331.6 + 15073.5) \cos 30^\circ \times 100 = 2,286,679.66 \text{ lb}$$

This force is resisted by the steel alone. Allowing a tensile stress of 20,000 psi in steel, then 114.3 sq.in of steel is required. That means 29 - #18 bars have to be provided. However in designing a large ring pretressing should always to be used. Assume the final stress of pretressed steel is of 120,000 psi



(a) From membrane theory (P.L.+L.L.)

(b) For edge forces acting on the shell

Fig. 6-1 Forces at boundary of the shell

$$A_s = \frac{2,286,679.66}{120,000} = 19.2 \text{ sq.in}$$

Which may be supplied by 25 wires, each 1 inch in diameter. An initial stress of 150,000 psi required to compensate for the assumed losses of 20 per cent. In Fig. 6-1 also show that the action of the ring in collection the tension forces. An increase in ring size and corresponding decrease the dome and wall hoop tension and bending moments.

The reinforcement arrangement for both the dome and the conical shell wall not only takes care the direct tension or compression stress, but also must provided the among of steel to resist the bending moment near the junction ef it. Hence it is beyond the scope of this report, there is no further discussion here.

Another important factor for stress analysis in the shell is the effect of the volume changes due to temperature. Suppose the temperature variation is 100°F, the radial displacement

$$H = -\epsilon t r_a = -0.0000065 \times 100 \times 100 = 0.065 \text{ ft}$$

The correspondent moment, meridian and hoop force due to this temperature change can be obtained simply from multplying this displacement with the value, by means of stiffness, which have already been obtained in previous calculations.

It is sometimes important to include the possible effects of wind load, eqrthquakes and blast loads. In analysis of the forces due to such loadings, generally, a loading pattern is assumed which can be expressed mathematically. All of these observations should be considered in desiging of shells. It is not intended that specific values given here be applied on the structure in

this report.

In analysis of the axisymmetric shells by the use of membrane theory, it is found to be quite simple. But generally this method is exact only for shells in which the practicular solution to the shell equations coincides with the membrane solution. In other words if the surface load produces bending of the shell the membrane theory is not accurate and may lead to great error in the solution. The analysis presented in this report combines both the membrane theory and bending analysis. Thought it is an approximate method in using finite difference method to solve the differential equations, yet till now it is the only method that can be applied in solving the parabolical shells. The numerical calculation in this report merely illustrate the principle involved. It is found that the solution is reliable and satisfactory.

NOTATIONS AND ABBREVIATIONS

Notation.

$D = \dfrac{Et^3}{12\,(1-\nu^2)}$ Bending rigidity of shell.

E = Young's modulus.

F = Flexbility.

H = Stiffness.

k = Constant defining the shape of paraboloid.

M'_{ϕ} = Meridional bending moment due to membrane theory.

M'_{θ} = Tangential bending moment due to membrane theory.

M_{ϕ} = Meridional bending moment.

M_{θ} = Tangential bending.

N_{ϕ} = Meridional membrane force.

N_{θ} = Hoop membrane force.

P_{ϕ} = External load per unit area acting in the direction of the tangent to the meridian.

P_z = External load per unit area acting inwards normal to the shell.

P_{θ} = External load per unit area in the direction tantent to the circular section.

p = Intensity of horizontal projecting load.

q = Intensity of uniform distributed load.

Q = Shear force.

r_0 = Radial distance from axis to point on the shell.

r_1 = Meridional radius of curvature.

r_2 = Circumferential radius of curvature.

R = End load.

s = Distance along the shell in the meridional direction.

t = Thickness of the shell.

U = $r_2 Q$

v = Angle of rotation of a tangent to a meridian.

w = Displacement normal to shell.

ν = Poisson's ratio.

ΔH = Radial displacement.

$\Delta \phi$ = Circumferential angle of meridian.

ϕ = Angle between axis of shell and normal to shell.

K = Modulus of fundation.

ϵ_ϕ = Strain in meridional direction.

ϵ_θ = Strain in horizontal direction.

Abbreviations.

psi = Pounds per sq. in.

lb = Pound(s)

in = Inches.

ft = Feet

sq = Squre

APPENDICES

```
C   C  PROPERTY OF THE DOME
       DIMENSION A(11,6),B(11,5)
       PI=3.1415926538
       DA=PI/60.
       A(1,1)=0.0
       DO 10 I=2,11
       J=I-1
    10 A(I,1)=A(J,1)+DA
       DO 20 I=1,11
       A(I,2)=SINF(A(I,1))
       A(I,3)=COSF(A(I,1))
       A(I,4)=1./A(I,3)
    20 A(I,5)=A(I,2)/A(I,3)
       A(1,6)=0.0
       DA=3.0
       DO 30 I=2,11
       J=I-1
    30 A(I,6)=A(J,6)+DA
       PUNCH 6
       PUNCH 5,((A(I,J),J=1,6),I=1,11)
       CK=SQRTF(3.)/600.
       DO 40 I=1,11
       B(I,1)=A(I,6)
       B(I,3)=A(I,4)/(2.*CK)
       B(I,2)=B(I,3)*A(I,4)**2
       B(I,4)=B(I,3)*A(I,2)
```

```
4  B(I,5)=CK*B(I,4)**2
   PUNCH 7
   PUNCH 8,((B(I,J),J=1,5),I=1,11)
 5 FORMAT(5F12.8,F6.2)
 6 FORMAT(6X,3HRAD,7X,4HSINX,8X,4HCOSX,8X,4HSECX,8X,4HTANX,6X,3HDEG/)
 7 FORMAT(//4X,3HDEG,7X,2HR1,13X,2HR2,14X,1HX,14X,1HY/)
 8 FORMAT(F8.2,4F14.8)
   STOP
   END
```

```
C   C MEMBRANE FORCE (DEAD LOAD)
      DIMENSION A(11,6),B(11,5),C(11,5)
      READ,((A(I,J),J=1,6),I=1,11)
      READ,((B(I,J),J=1,5),I=1,11)
      CK=SQRT(3.)/600.
      Q=75.
      PI=3.14159265
      I=1
      C(I,1)=0.0
      C(I,2)=0.0
      C(I,3)=-Q/(4.*CK)
      C(I,4)=C(I,3)
      DO 10 I=2,11
      J=I
      C(I,1)=B(J,1)
      W=1.+4.*CK**2*B(I,4)**2
      S=SQRT(W)
      C(I,2)=PI*Q*(S**3-1.)/(6.*CK**2)
      C(I,3)=-C(I,2)*S/(4.*PI*CK*B(J,4)**2)
   18 C(I,4)=C(I,2)/(2.*PI*B(J,2)*A(J,2)**2)
   10 C(I,4)=C(I,4)-Q*B(J,4)*A(J,3)/A(J,2)
      PUNCH 15
   17 PUNCH 16,((C(I,J),J=1,4),I=1,11)
   15 FORMAT (//4X,3HDEG,13X,1HR,17X,2HNM,16X,2HNR)
   16 FORMAT (F8.2,3F18.4)
      STOP
      END
```

```
C   C   MEMBRANE FORCES DUE TO SNOW LOAD
        DIMENSION A(11,6),B(11,5),D(11,4)
        READ,((A(I,J),J=1,6),I=1,11)
        READ,((B(I,J),J=1,5),I=1,11)
        CK=SQRT(3.)/600.
        PJ=30.
        PI=3.14159265
        I=1
        D(I,1)=0.0
        D(I,2)=0.0
        D(I,3)=-PJ/(4.*CK)
        D(I,4)=D(I,3)
        DO 10 I=2,11
        D(I,1)=B(I,1)
        W=1.+4.*CK**2*B(I,4)**2
        S=SQRT(W)
        D(I,2)=PI*PJ*B(I,4)**2
        D(I,3)=-D(I,2)/(2.*P1*B(I,4)*A(I,2))
     10 D(I,4)=D(I,2)/(2.*PI*B(I,2)*A(1,2)**2)-PJ*B(I,3)*A(I,3)**2
        PUNCH 15
        PUNCH 16,((D(I,J),J=1,4),I=1,11)
     15 FORMAT (//4X,3HDEG,13X,1HR,17X,3HNMP,16X,3HNRP)
     16 FORMAT (F8.2,3F18.4)
        STOP
        END
```

```
C C   DISPLACEMENT FROM MEMBRANE THEORY
      DIMENSION A(11,6),B(11,5),C(11,4),D(11,4),G(11,13)
      READ,((A(I,J),J=1,6),I=1,11)
      READ,((B(I,J),J=1,5),I=1,11)
      READ,((C(I,J),J=1,4),I=1,11)
      V=0.2
      T=0.5
      G(1,11)=0.0
      G(1,12)=0.0
      G(1,13)=0.0
      DO 10 I=2,11
    9 G(I,1)=B(I,1)
      G(I,2)=B(I,3)*A(I,2)/T
      G(I,3)=1./(B(I,2)*T*A(I,5))
      G(I,4)=G(I,3)*(C(I,3)*(B(I,2)+V*B(I,3))-C(I,4)*(B(I,3)+V*B(I,2)))
      G(I,5)=-2./(A(I,2)**2*A(I,2))+A(I,4)**2/A(I,2)+2.*A(I,3)/A(I,2)
      G(I,6)=2./(A(I,5)**2*A(I,5))-3.*A(I,4)*A(I,5)
      G(I,7)=G(I,5)+G(I,6)
      G(I,8)=-A(I,4)**2/(A(I,2)*A(I,5))+3.*A(I,4)**2*A(I,4)**2
      G(I,9)=A(I,4)/(A(I,2)**2*A(I,2))-A(I,4)/A(I,2)
      G(I,10)=G(I,7)-V*(G(I,8)+G(I,9)+1./A(I,2)**2*A(I,3))
      G(I,11)=B(I,1)
      G(I,12)=G(I,2)*(C(I,4)-V*C(I,3))
   10 G(I,13)=(G(I,4)-G(I,10))/B(I,2)
      PUNCH 5
      PUNCH 6,((G(I,J),J=11,13),I=1,11)
```

```
5 FORMAT (//4X,3HDEG,8X,6HE*D(H),14X,7HE*D(AN)
6 FORMAT (F8.2,2F18.6)
  STOP
  END
```

```
C   C   COEFFECIENT MATRIX OF AX=G
        DIMENSION A(11,6),B(22,22)
        READ,((A(I,J),J=1,6),I=1,11)
        DA=3.1415926536/60.
        DD=DA**2
        DO 10 I=1,22
        DO 10 J=1,22
        D4=4.*DD
        CK=SQRT(3.)/600.
        G=6.*(1.-0.25**2)*DD/CK
    10  B(I,J)=0.
        B(1,1)=-(2.+DD/(A(2,2)*A(2,3))**2)
        B(1,2)=-0.5*(DD*A(2,4)**5)/(2.*CK)
        B(1,3)=1.+DA*(-2.*A(2,5)+1./A(2,5))/2.
        B(2,1)=G*A(2,4)**5/(0.5**3)
        B(2,2)=B(1,1)
        B(2,4)=B(1,3)
        DO 20 I=3,19,2
        J=I+1
        K=I+2
        L=I+3
        N=(I+3)/2
        M=I-1
        KL=I-2
        B(I,KL)=1.+(2.*A(N,5)-1./A(N,5))*DA/2.
        B(I,I)=-(2.+DD/(A(N,2)*A(N,3))**2)
```

```
      B(I,J)=-0.5*DD*A(N,4)**5/(2.*CK)
      B(I,K)=1.+DA*(-2.*A(N,5)+1./A(N,5))/2.
      B(J,M)=B(I,KL)
      B(J,I)=(G*A(N,4)**5)/0.125
      B(J,J)=B(I,I)
   20 B(J,L)=B(I,K)
      DK=A(11,3)**2*A(11.2)/(2.*0.5*DA)
      DO 30 I=1,22
      DO 30 J=1,22
      B(21,17)=DK
      B(21,19)=0.2/(0.5*A(11,2))
      B(21,21)=-DK
   30 B(22,20)=1.0
      PUNCH 6,((B(I,J),J=1,22),I=1,22)
    6 FORMAT (4F16.8)
      STOP
      END
```

```
C  C  INVERSION OF MATRIX A
      DIMENSION A(22,22),B(22),C(22)
   12 FORMAT (3E24.8)
      READ,N
      NN=N-1
   11 FORMAT (3F24.0)
   10 READ,((A(I,J),J=1,N),I=1,N)
      PUNCH 5,((A(I,J),J=1,15),I=1,15)
    5 FORMAT (10F7.1)
      A(1,1)=1.0/A(1,1)
      DO 110 M=1,NN
      K=M+1
   50 DO 60 I=1,M
      B(I)=0.
      DO 60 J=1,M
   60 B(I)=B(I)+A(I,J)*A(J,K)
      D=0.0
      DO 70 I=1,M
   70 D=D+A(K,I)*B(I)
      D=-D+A(K,K)
      A(K,K)=1./D
      DO 80 I=1,M
   80 A(I,K)=-B(I)*A(K,K)
      DO 90 J=1,M
      C(J)=0.
      DO 90 I=1,M
```

```
 90 C(J)=C(J)+A(K,I)*A(I,J)
    DO 100 J=1,M
100 A(K,J)=-C(J)*A(K,K)
    DO 110 I=1,M
    DO 110 J=1,M
110 A(I,J)=A(I,J)-B(I)*A(K,J)
    DO 170 I=1,N
170 PUNCH 12,(A(I,J),J=1,N)
    STOP
    END
```

```
C   C  FORCES DUE TO BOUNDARY DISP. (D(H)=1,D(AN)=0 AND D(H)=0,D(AN)=1)
       DIMENSION A(11,6),B(11,5),C(12),D(11,3)
       READ,((A(I,J),J=1,6),I=1,11)
       READ,((B(I,J),J=1,5),I=1,11)
       C(1)=0.0
       DO 16 LM=1,2
    11 READ,(C(I),I=2,12)
       DA=3.1415926538/60.
       D(1,1)=0.0
       D(1,2)=0.0
       D(1,3)=0.0
       DO 10 I=2,11
       M=I-1
       L=I+1
       D(I,1)=B(I,1)
       D(I,2)=-C(I)/(B(I,3)*A(I,5))
    10 D(I,3)=-(C(L)-C(M))/(2.*DA*B(I,2))
       PUNCH 5
       PUNCH 6,((D(I,J),J=1,3),I=1,11)
    16 CONTINUE
     5 FORMAT (//4X,3HDEG,15X,4HN(M),15X,5HN(AN))
     6 FORMAT (F8.2,2F20.8)
       STOP
       END
```

```
C   C  MOMENT DUE TO BOUNDARY DISP. (D(H)=1.,D(AN)=0,AND D(H)=0,D(AN)=1.)
       DIMENSION A(11,6),B(11,5),C(12),D(11,5)
       READ,((A(I,J),J=1,6),I=1,11)
       READ,((B(I,J),J=1,5),I=1,11)
       C(1)=0.0
       DO 16 LM=1,2
    11 READ12,(C(I),I=2,12)
       DA=3.1415926536/60.
       V=0.2
       DD=-0.5**3./(12.*(1.-0.2**2))
       DO 10 I=2,11
       L=I+1
       M=I-1
       D(1,3)=0.0
       D(1,4)=0.0
       D(1,5)=0.0
       D(I,1)=(C(L)-C(M))/(2.*DA*B(I,2))
       D(I,2)=V*C(I)/(A(I,5)*B(I,3))
       D(I,3)=B(I,1)
       D(I,4)=DD*(D(I,2)+D(I,1))
    10 D(I,5)=DD*(D(I,2)/V+V*D(I,1))
       PUNCH 5
       PUNCH 6,((D(I,J),J=3,5),I=1,11)
     5 FORMAT (4X,3HDEG,13X,4HM(M),17X,5HM(AN)
     6 FORMAT (F8.2,2F20.8)
    16 CONTINUE
```

```
12 FORMAT (E24.8)
   STOP
   END
```

```
C   C  MEMBRANE FORCES DUE TO DEAD LOAD
       DIMENSION A(11,3)
       PI=3.1415926538
       DA=5.
       DP=115.45
       DL=65.45
       DQ=100.
       AA=COSF(1.04719744)
       AC=SINF(1.04719744)
       AB=SINF(1.04719744)/SINF(1.04719744)
       DO 10 I=2,11
       J=I-1
       A(1,1)=65.45
    10 A(I,1)=A(J,1)+DA
       DO 20 I=1,11
       A(I,2)=-DQ*(DP**2-A(I,1)**2)/(2.*A(I,1)*AA)
    20 A(I,3)=DQ*A(I,1)*AB*AC
       PUNCH 5
       PUNCH 6,((A(I,J),J=1,3),I=1,11)
     5 FORMAT (/4X,1HY,18X,4HN(Y),18X,5HN(AN))
     6 FORMAT (F8.2,2F24.8)
       STOP
       END
```

```
C   C  MEMBRANE FORCES DUE TO DOME LOAD P
       DIMENSION A(11,3)
       PI=3.1415926538
       R=-2.*PI*(100.+1.75)*3.5*3.5*150.-3559931.
       DA=5.
       A(1,1)=65.45
       AA=COSF(1.04719744)
       AC=SINF(1.04719744)
       AB=SINF(1.04719744)/SINF(1.04719744)
       DO 10 I=2,11
       J=I-1
    10 A(I,1)=A(J,1)+DA
       DO 20 I=1,11
       A(I,2)=R/(PI*A(I,1)*2.*AC*AA)
    20 A(I,3)=-R*AB/(2.*PI)
       PUNCH 5
       PUNCH 6,((A(I,J),J=1,3).I=1,11)
     5 FORMAT (/4X,1HY,18X,4HN(Y),18X,5HN(AN))
     6 FORMAT (F8.2,2F24.8)
       STOP
       END
```

```
C   C  DISPLACEMENT AND ROTATION DUE TO MEMEBRANE THEORY
       DIMENSION A(14,4),B(14,4)
       T=0.6666667
       Q=100.
       PI=3.1415926538
       DA=5.
       DL=65.45
       V=0.2
       R=+2.*PI*(100.+1.75)*3.5*3.5*150.+3559931.
       AA=COSF(1.04719744)
       AB=SINF(1.04719744)
       AC=AA/AB
       DO 10 I=2,11
       J=I-1
       A(1,1)=65.45
    10 A(I,1)=A(J,1)+DA
       DO 20 I=1,11
       AD=Q/T
       AE=AB*AC*A(I,1)
       AF=1./(2.*A(I,1)*AA)
       A(I,2)=AD*A(I,1)*AB*(AE+AF*V*(115.5**2-A(I,1)**2))
    20 A(I,3)=AD*AC*AF*(A(I,1)*(A(I,1)-V)-115.45**2*(1.+V))
       PUNCH 6
       PUNCH 7,((A(I,J),J=1,3),I=1,11)
       DO 30 I=1,11
       B(I,1)=A(I,1)
```

```
      B(I,2)=R*A(I,1)*AB*(AC+V/(A(I,1)*AB*AA))/(2.*PI*T)
      B(I,3)=R*AC*((V-1.)/(A(I,1)*AA*AB)-V*AC)
   30 B(I,3)=B(I,3)/(2.*PI*T)
      PUNCH 6
      PUNCH 7,((B(I,J),J=1,3),I=1,11)
    6 FORMAT (/4X,1HY,16X,4HE*DH,18X,4HE*DA)
    7 FORMAT (F8.2,2F24.8)
      STOP
      END
```

```
C  C  COEFFICIENT MATRIX OF AX=G
      DIMENSION A(15,15),B(15)
      V=0.2
      PI=3.1415926538
      COT=COSF(1.04719744)/SINF(1.04719744)
      DA=5.
      DB=0.666667
      B(1)=55.45
      DO 10 I=2,15
      J=I-1
   10 B(I)=B(J)+DA
      DO 20 I=1,15
      DO 20 J=1,15
   20 A(I,J)=0.0
      A(2,2)=-1.0
      A(2,4)=1.0
      A(3,3)=1.0
      A(14,12)=-1.0
      A(14,14)=1.0
      A(15,13)=1.0
      DO 30 I=3,13
      A(1,1)=(B(3)/DA-1.)/DA**3
      A(1,2)=(-4.*B(3)/DA+2.)/DA**3
      A(1,3)=(B(3)/DA**4+2.*(COT)**2*(1.-V**2)/(DB**2*B(3)))*6.
      A(1,4)=(2.*B(3)/DA+1.)*(-2.)/DA**3
   30 A(1,5)=(B(3)/DA+1.)/DA**3
```

```
      DO 40 I=4,13
      N=I+1
      K=I+2
      M=I-1
      L=I-2
      A(I,L)=(B(I)/DA-1.)/DA**3
      A(I,M)=(-4.*B(I)/DA+2.)/DA**3
      A(I,I)=(B(I)/DA**4+2.*(COT)**2*(1.-V**2)/(DB**2*B(I)))*6.
      A(I,N)=(2.*B(I)/DA+1.)*(-2.)/DA**3
   40 A(I,K)=(B(I)/DA+1.)/DA**3
      PUNCH 5,((A(I,J),J=1,15),I=1,15)
    5 FORMAT(4F18.8)
      STOP
      END
```

```
C   C   CALCULATION OF Z-VECTOR
        DIMENSION A(15,15),B(22),C(20,5)
     12 FORMAT (3E24.8)
        DO 15 I=1,15
     15 READ 12,(A(I,J),J=1,15)
        C(1,1)=55.45
        DA=5.
        DO 180 I=2,15
        J=I-1
    180 C(I,1)=C(J,1)+DA
        DO 200 K=1,4
        READ,(B(I),I=1,15)
        DO 120 I=1,15
        C(I,2)=0.0
        DO 120 J=1,15
    120 C(I,2)=C(I,2)+A(I,J)*B(J)
        PUNCH 130
    130 FORMAT (/4X,1HY,12X,8HZ-VECTOR)
    140 PUNCH 6,((C(I,J),J=1,2),I=1,15)
      6 FORMAT (F8.2,F24.8)
    200 CONTINUE
        STOP
        END
```

```
C   C  FORCES DUE TO EDGE EFFECT
       DIMENSION A(15,2),B(15,4),C(15,3)
       TAN=SINF(1.04719744)/COSF(1.04719744)
       V=0.2
       T=0.666667
       DA=5.
       R=T**3/(12.*(1.-V**2)*COSF(1.0471944)**3)
       DO 200 K=1,4
       READ,((A(I,J),J=1,2),I=1,15)
       DO 10 I=3,13
       L=I-2
       M=I-1
       N=I+1
       LM=I+2
       AA=-A(L,2)*A(I,1)/DA**3+A(M,2)*(A(I,1)/DA+1.)/DA**2
       AA=AA-A(I,2)/DA**2+A(N,2)*(1.-A(I,1)/DA)/DA**2
       AA=AA+A(LM,2)*A(I,1)/(2.*DA**3)
       B(I,1)=A(I,1)
       B(I,2)=R*AA
       B(I,3)=B(I,2)*TAN
    10 B(I,4)=T*A(I,2)/(A(I,1)*TAN)
       PUNCH 5
       PUNCH 7,((B(I,J),J=1,4),I=3,13)
       DO 20 I=3,13
       C(I,1)=A(I,1)
       C(I,2)=-R*(A(M,2)-2.*A(I,2)+A(N,2))/DA**2
```

```
2( C(I,3)=V*C(I,2)
   PUNCH 4
   PUNCH 6,((C(I,J),J=1,3),I=3,13)
 4 FORMAT (//4X,1HY,18X,4HM(Y),20X,5HM(AN))
 5 FORMAT (//4X,1HY,16X,1HQ,18X,4HN(Y),16X,5HN(AN))
 6 FORMAT (F8.2,2F24.8)
 7 FORMAT (F8.2,3F20.8)
200 CONTINUE
   STOP
   END
```

```
C   C   CALCULATION OF FLIXBILITY MATRIX (WALL)
        DIMENSION A(22,22),B(22),C(22),E(22)
     11 FORMAT (3F24.8)
     12 FORMAT (3E24.8)
        READ,N
        NN=N-1
        DO 10 I=1,4
        AA=SINF(1.04719744)
        A(1,1)=-0.53652068*AA
        A(1,2)=1.2812699*AA
        A(1,3)=0.00194688*AA
        A(1,4)=0.00332947*AA
        A(2,1)=0.00915366
        A(2,2)=-0.0430242
        A(2,3)=0.03292389
        A(2,4)=0.00010094
        A(3,1)=-0.00165074*AA
        A(3,2)=-0.004603333*AA
        A(3,3)=1.1707872*AA
        A(3,4)=3.3840397*AA
        A(4,1)=-0.02376767
        A(4,2)=-0.0430242
        A(4,3)=0.00000257
     10 A(4,4)=0.00010094
        A(1,1)=1.0/A(1,1)
        DO 110 M=1,NN
```

```
      K=M+1
   50 DO 60 I=1,M
      B(I)=0.
      DO 60 J=1,M
   60 B(I)=B(I)+A(I,J)*A(J,K)
      D=0.0
      DO 70 I=1,M
   70 D=D+A(K,I)*B(I)
      D=-D+A(K,K)
      A(K,K)=1./D
      DO 80 I=1,M
   80 A(I,K)=-B(I)*A(K,K)
      DO 90 J=1,M
      C(J)=0.
      DO 90 I=1,M
   90 C(J)=C(J)+A(K,I)*A(I,J)
      DO 100 J=1,M
  100 A(K,J)=-C(J)*A(K,K)
      DO 110 I=1,M
      DO 110 J=1,M
  110 A(I,J)=A(I,J)-B(I)*A(K,J)
      DO 170 I=1,N
  170 PUNCH 11,(A(I,J),J=1,N)
      DO 20 I=1,2
      E(1)=150.**1.5*33.*SQRTF(4000.)
   20 E(2)=150.**1.5*33.*SQRTF(4000.)*144.
```

```
      PUNCH 5
      PUNCH 6,(F(I),I=1,2)
    5 FORMAT (//4X,7HE-VALUE)
    6 FORMAT (2F24.8)
      STOP
      END
```

```
C   C  CALCULATION OF X VALUE
       DIMENSION A(22,22),B(22),C(20,2),E(11)
    11 FORMAT (3F24.8)
    12 FORMAT (3E24.8)
       READ,N
       NN=N-1
    10 READ,((A(I,J),J=1,N),I=1,N)
       PUNCH 15
    15 FORMAT (//4X,3H(1),1X,4HCOEF,1X,6HMATRIX,1X,1HA)
       PUNCH 11,((A(I,J),J=1,N),I=1,N)
       A(1,1)=1.0/A(1,1)
       DO 110 M=1,NN
       K=M+1
    50 DO 60 I=1,M
       B(I)=0.
       DO 60 J=1,M
    60 B(I)=B(I)+A(I,J)*A(J,K)
       D=0.0
       DO 70 I=1,M
    70 D=D+A(K,I)*B(I)
       D=-D+A(K,K)
       A(K,K)=1./D
       DO 80 I=1,M
    80 A(I,K)=-B(I)*A(K,K)
       DO 90 J=1,M
       C(J)=0.
```

```
      DO 90 I=1,M
   90 C(J)=C(J)+A(K,I)*A(I,J)
      DO 100 J=1,M
  100 A(K,J)=-C(J)*A(K,K)
      DO 110 I=1,M
      DO 110 J=1,M
  110 A(I,J)=A(I,J)-B(I)*A(K,J)
      PUNCH 14
   14 FORMAT (//4X,3H(2),1X,9HA-INVERSE)
      DO 170 I=1,N
  170 PUNCH 11,(A(I,J),J=1,N)
      WTH=67431871.4
      WTA=-89831.86
      WBH=38468895.2
      WBA=-124701.59
      AA=815.
      AD=1421.
      AJ=800.
      DDH=-1484699.
      DDA=-49.7
      XD=-11331.6134
      XWT=-15673.541
      NWB=-54629.318
      DO 20 I=1,6
      B(1)=-DDH-1.72*XD
      B(2)=-DDA+AD*XD
```

```
      B(3)=-WTH-0.877*AA*XWT
      B(4)=-WTA-AE*0.6*XWT
      B(5)=-WBH
   20 B(6)=-WBA
      PUNCH 5
      PUNCH 6,(B(I),I=1,6)
    5 FORMAT (//4X,1X,3H(2),1X,5HCONST,1X,6HMATRIX)
    6 FORMAT (F24.8)
      DO 120 I=1,6
      C(I)=0.0
      DO 120 J=1,6
  120 C(I)=C(I)+A(I,J)+A(I,J)*B(J)
      PUNCH 130
  130 FORMAT (//4X,3H(4),1X,1HX,1X,5HFORCE)
      PUNCH 6,(C(I),I=1,6)
      STOP
      END
```

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STRESS ANALYSIS OF A SHELL STRUCTURE

by

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B. Sc. Taiwan Cheng Kung University, 1961

AN ABSTRACT OF A MASTER'S REPORT

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requirement for the degree

MASTER OF SCIENCE

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KANSAS STATE UNIVERSITY
Manhattan, Kansas

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The purpose of this report was to introduce the design of thin shells by (1) deriving the membrane equations for the elastic analysis of shells of the form of a surface of revolution and loaded symmetrically with respect to the axis (2) by describing the physical behavior of a well defined system. In order to illustrate the formula derived, the dome structure is presented.

Due to membrane theory the stresses analysis of a shell structure sometimes is not true. Timoshenko develops the general homogenous equations for an axisymmetric shells. To solve a shell problem a membrane solution is superimposed upon the solution of the homogeneous equations. The complete procedure in analysis of a shell structure can be outlined as follows:

1. Calculation of forces due to membrane theory.

2. Calculation of the displacements at the boundary of the shell from the membrane theory.

3. The corrections correspond to unit edge effects derived from the solution of the homogeneous equations commonly referred to as the bending theory.

4. Compatibility is obtained by determining the size of the corrections required to remove the errors in the membrane theory.

In design a shell structure not only consider the effects due to the symmetrical surface load and dead load, but also need to consider the effect due to wind load, temperature etc. However, it is beyond the scope of this report, the author only mentioned it in the part of conclusion and discussions.